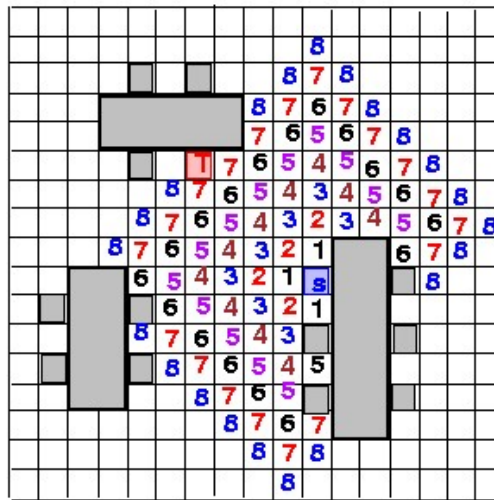
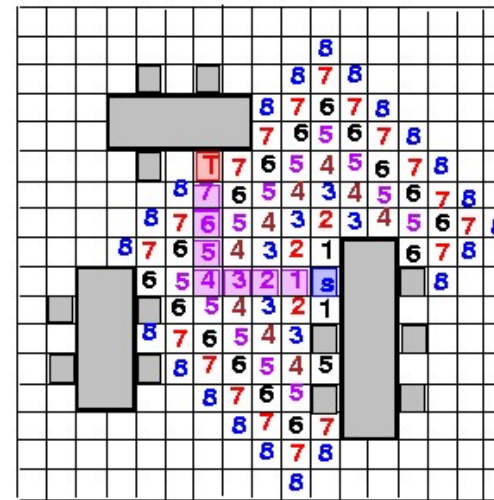


# Unit 6: Maze (Area) and Global Routing

- Course contents
  - Routing basics
  - Maze (area) routing
  - Global routing

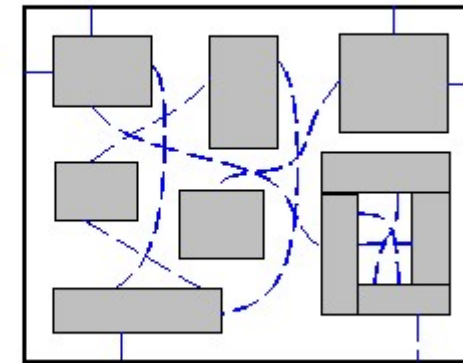
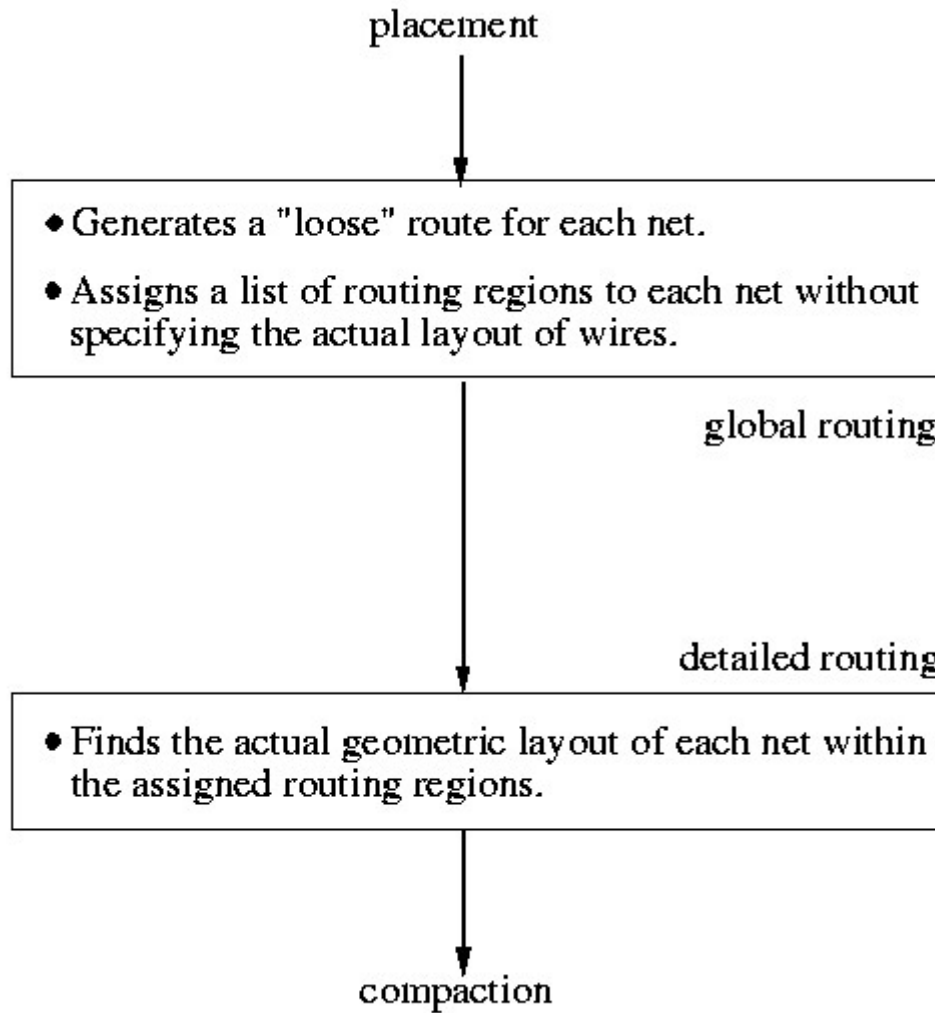


Filling

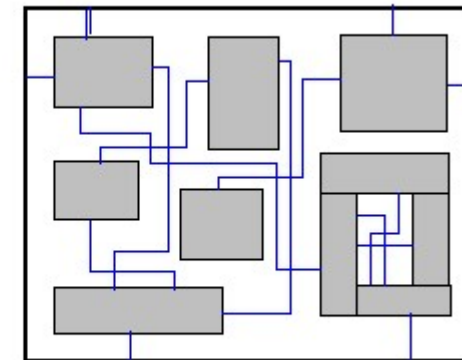


Retrace

# Routing



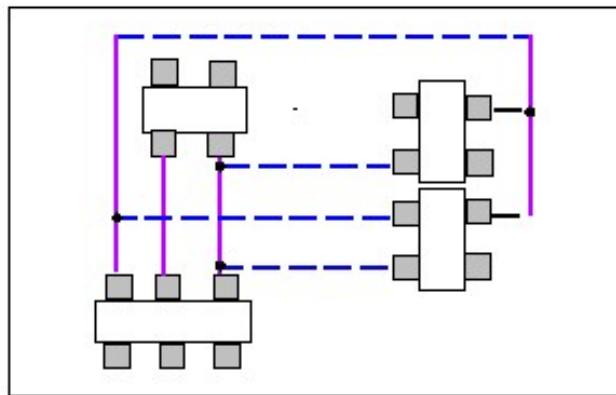
Global routing



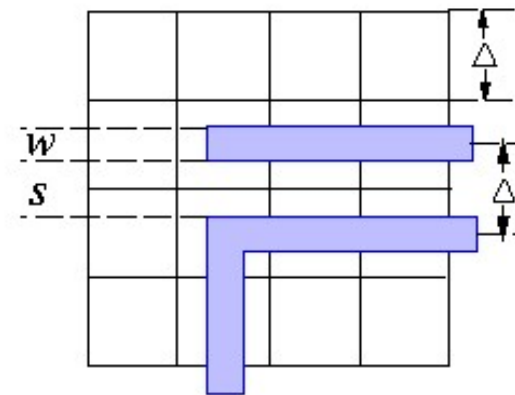
Detailed routing

# Routing Constraints

- 100% routing completion + area minimization, under a set of constraints:
  - Placement constraint: usually based on fixed placement
  - Number of routing layers
  - Geometrical constraints: must satisfy design rules
  - Timing constraints (performance-driven routing): must satisfy delay constraints
  - Crosstalk?
  - Process variations?

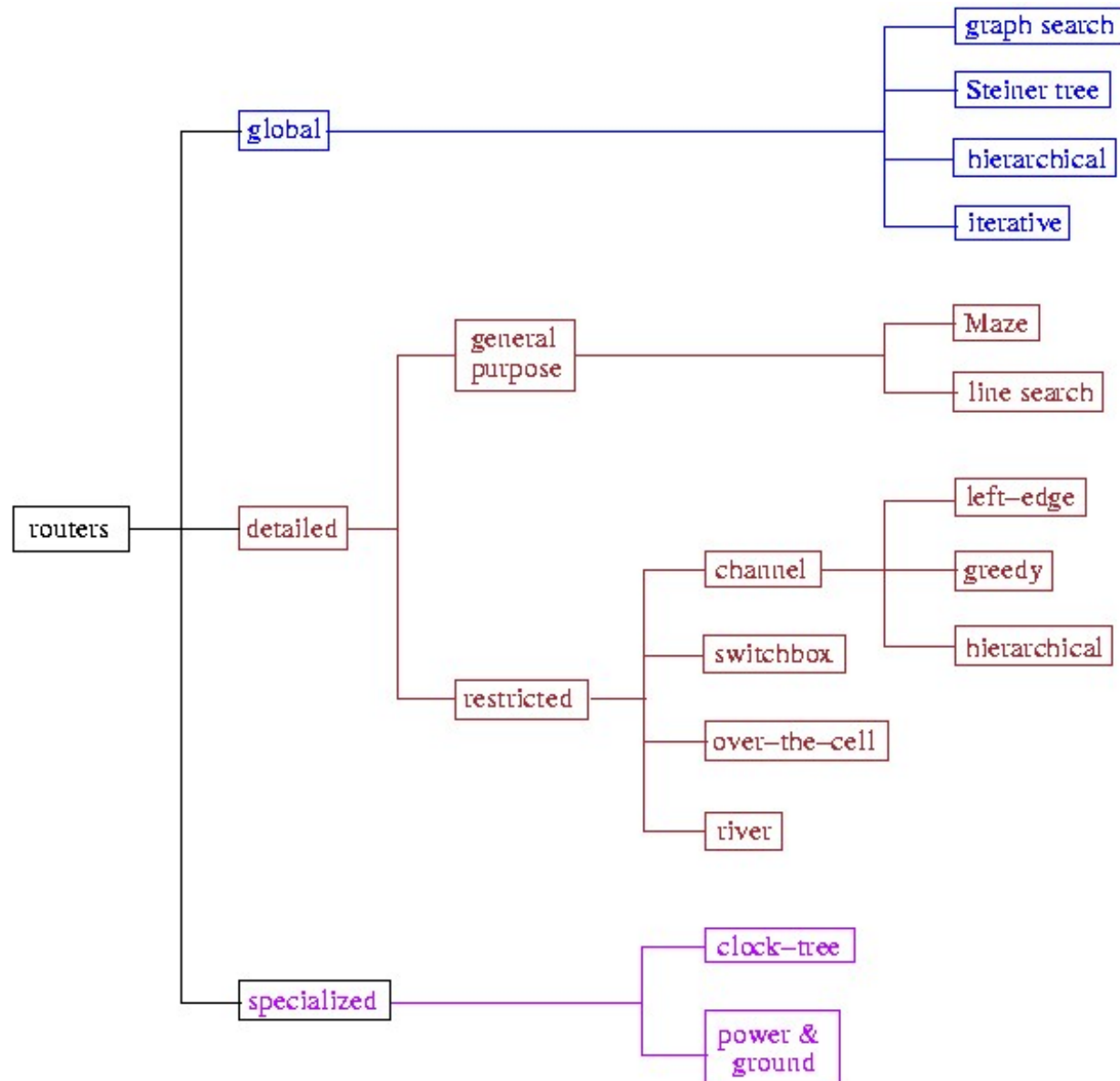


*Two-layer routing*



*Geometrical constraint*

# Classification of Routing



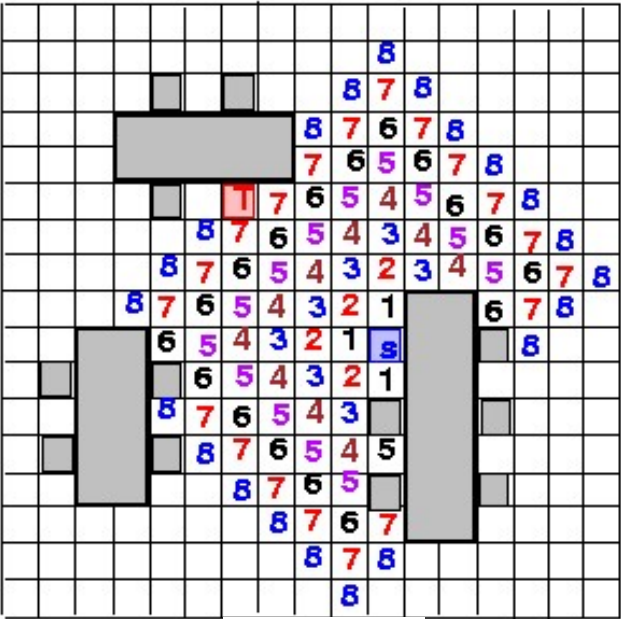
# Maze Router: Lee Algorithm

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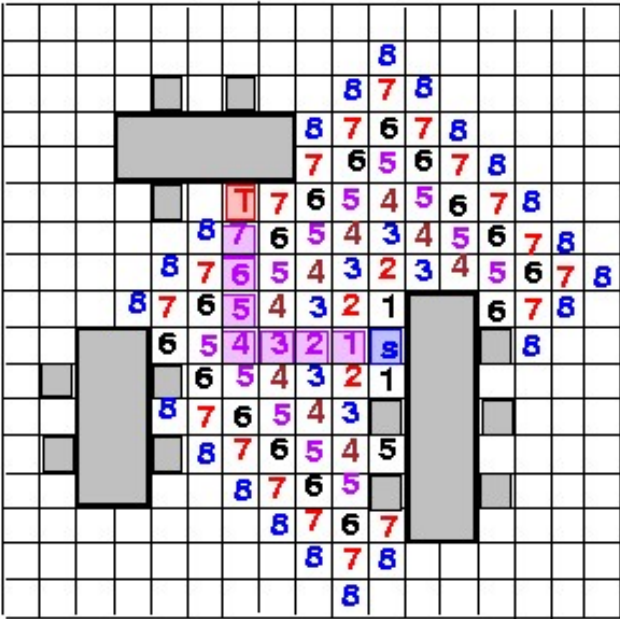
- Lee, “An algorithm for path connection and its application,” *IRE Trans. Electronic Computer*, EC-10, 1961.
- Discussion mainly on single-layer routing
- **Strengths**
  - Guarantee to find connection between 2 terminals if it exists.
  - Guarantee minimum path.
- **Weaknesses**
  - Requires large memory for dense layout.
  - Slow.
- Applications: global routing, detailed routing

# Lee Algorithm

- Find a path from  $S$  to  $T$  by “wave propagation”.



Filling



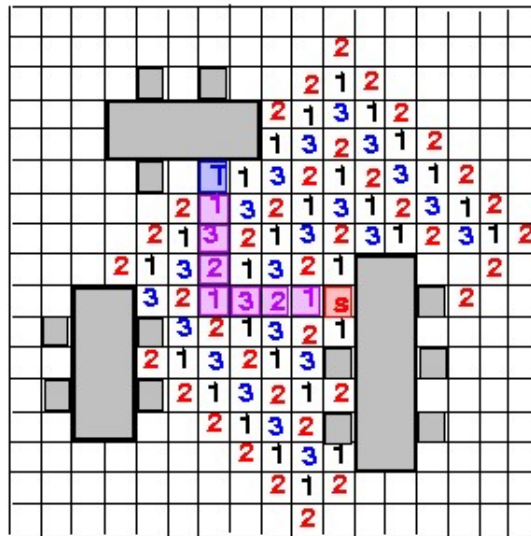
Retrace

- Time & space complexity for an  $M \times N$  grid:  $O(MN)$  (huge!)

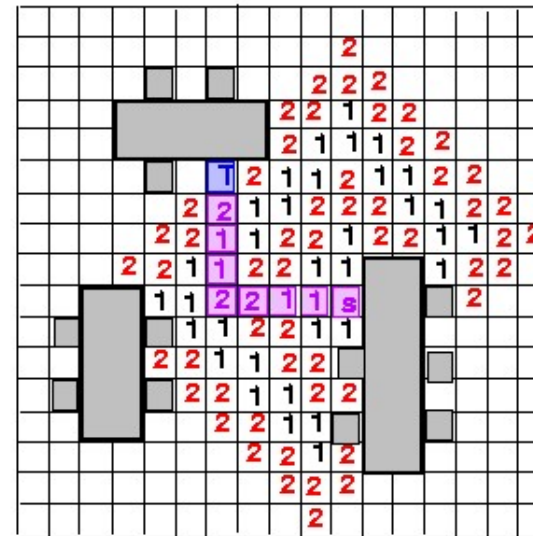


# Reducing Memory Requirement

- Akers's Observations (1967)
  - Adjacent labels for  $k$  are either  $k-1$  or  $k+1$ .
  - Want a labeling scheme such that each label has its preceding label different from its succeeding label.
- Way 1: coding sequence 1, 2, 3, 1, 2, 3, ...; states: 1, 2, 3, *empty*, *blocked* (3 bits required)
- Way 2: coding sequence 1, 1, 2, 2, 1, 1, 2, 2, ...; states: 1, 2, *empty*, *blocked* (need only 2 bits)



Sequence: 1, 2, 3, 1, 2, 3, ...



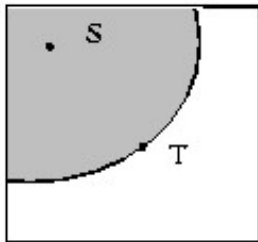
Sequence: 1, 1, 2, 2, 1, 1, 2, 2, ...

# Reducing Running Time

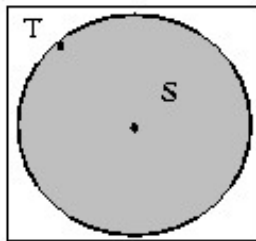
---

- Starting point selection: Choose the point farthest from the center of the grid as the starting point.
- Double fan-out: Propagate waves from both the source and the target cells.
- Framing: Search inside a rectangle area 10--20% larger than the bounding box containing the source and target.
  - Need to enlarge the rectangle and redo if the search fails.

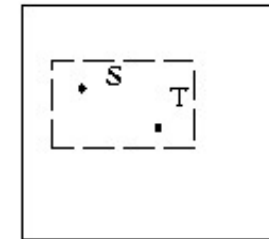
starting point selection



double fan-out

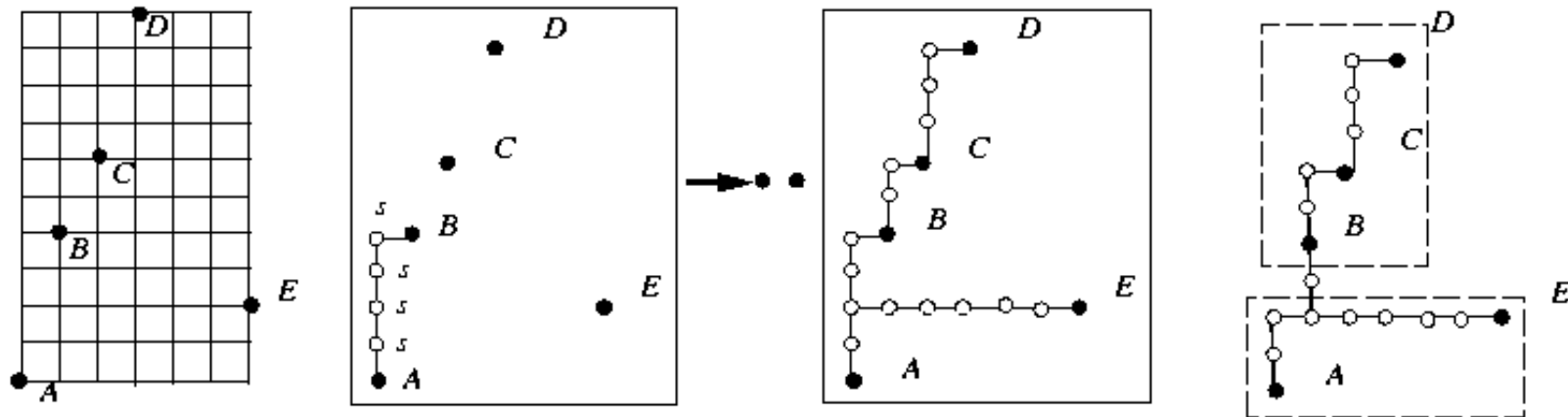


framing



# Connecting Multi-Terminal Nets

- Step 1: Propagate wave from the source  $s$  to the closest target.
- Step 2: Mark ALL cells on the path as  $s$ .
- Step 3: Propagate wave from ALL  $s$  cells to the other cells.
- Step 4: Continue until all cells are reached.
- Step 5: Apply heuristics to further reduce the tree cost.



# Routing on a Weighted Grid

- Motivation: finding more desirable paths
- $weight(grid\ cell) = \# \text{ of unblocked grid cell segments} - 1$



# A Routing Example on a Weighted Grid

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
|   |   |   |   |   |   |   |   |   | 2 |
|   |   | 1 | 2 | 2 | 2 | 2 | 2 | 1 | 3 |
|   |   | 1 | 3 | 3 | 3 | 3 | 2 | S | 2 |
| 2 | 1 | T | 2 | 3 | 3 | 3 | 3 | 2 | 3 |
| 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

*initialize cell weights*

|  |  |   |  |  |   |   |   |   |   |
|--|--|---|--|--|---|---|---|---|---|
|  |  |   |  |  |   |   |   |   |   |
|  |  |   |  |  |   |   |   |   |   |
|  |  |   |  |  |   |   | 3 | 1 | 4 |
|  |  |   |  |  | 5 | 2 | S | 2 |   |
|  |  | T |  |  |   | 5 | 2 | 5 |   |
|  |  |   |  |  |   |   | 5 |   |   |

*wave propagation*

|  |  |  |  |  |  |  |  |    |    |    |    |    |   |   |   |
|--|--|--|--|--|--|--|--|----|----|----|----|----|---|---|---|
|  |  |  |  |  |  |  |  | 13 | 11 | 9  |    |    |   |   |   |
|  |  |  |  |  |  |  |  |    |    | 6  |    |    |   |   |   |
|  |  |  |  |  |  |  |  | 11 | 9  | 7  | 5  | 3  | 1 | 4 |   |
|  |  |  |  |  |  |  |  | 15 | 14 | 11 | 8  | 5  | 2 | S | 2 |
|  |  |  |  |  |  |  |  | T  | 16 | 14 | 11 | 8  | 5 | 2 | 5 |
|  |  |  |  |  |  |  |  |    |    | 17 | 14 | 11 | 8 | 5 | 8 |

|  |  |  |  |  |  |  |  |    |    |    |    |    |   |   |   |
|--|--|--|--|--|--|--|--|----|----|----|----|----|---|---|---|
|  |  |  |  |  |  |  |  | 15 | 13 | 11 | 9  |    |   |   |   |
|  |  |  |  |  |  |  |  |    |    |    | 6  |    |   |   |   |
|  |  |  |  |  |  |  |  | 12 | 11 | 9  | 7  | 5  | 3 | 1 | 4 |
|  |  |  |  |  |  |  |  | 15 | 14 | 11 | 8  | 5  | 2 | S | 2 |
|  |  |  |  |  |  |  |  | 15 | 16 | 14 | 11 | 8  | 5 | 2 | 5 |
|  |  |  |  |  |  |  |  |    | 19 | 17 | 14 | 11 | 8 | 5 | 8 |

*first wave reaches the target*

|  |  |  |  |  |  |  |  |    |    |    |    |    |   |   |   |   |
|--|--|--|--|--|--|--|--|----|----|----|----|----|---|---|---|---|
|  |  |  |  |  |  |  |  | 17 | 15 | 13 | 11 | 9  |   |   |   |   |
|  |  |  |  |  |  |  |  |    |    |    |    | 6  |   |   |   |   |
|  |  |  |  |  |  |  |  | 12 | 11 | 9  | 7  | 5  | 3 | 1 | 4 |   |
|  |  |  |  |  |  |  |  | 13 | 14 | 11 | 8  | 5  | 2 | S | 2 |   |
|  |  |  |  |  |  |  |  | 16 | 15 | 16 | 14 | 11 | 8 | 5 | 2 | 5 |
|  |  |  |  |  |  |  |  | 17 | 19 | 17 | 14 | 11 | 8 | 5 | 8 |   |

*finding other paths*

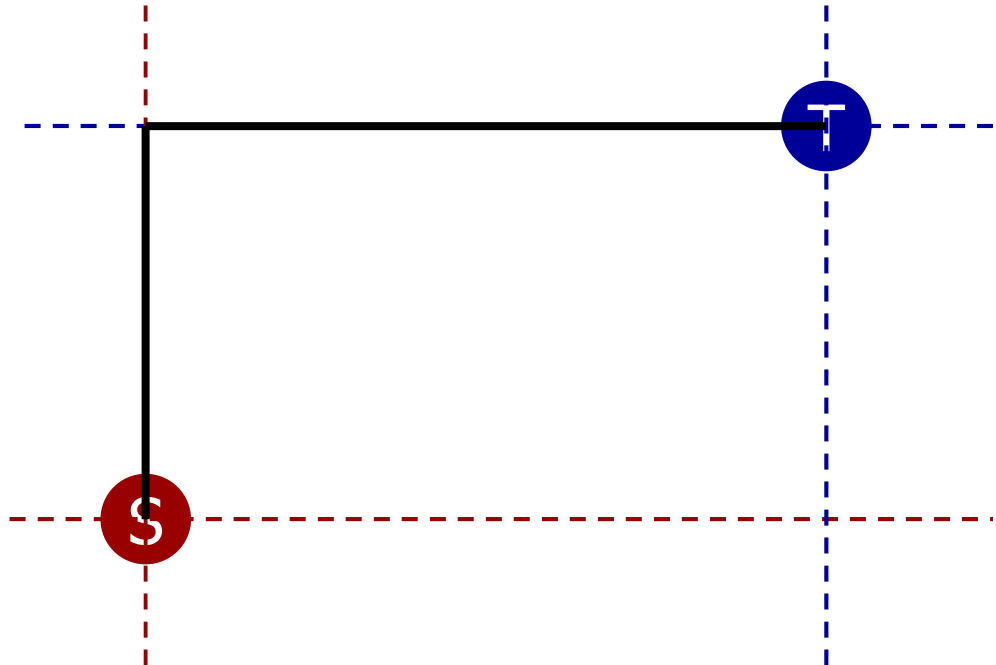
|  |  |  |  |  |  |  |  |    |    |    |    |    |    |   |   |   |   |
|--|--|--|--|--|--|--|--|----|----|----|----|----|----|---|---|---|---|
|  |  |  |  |  |  |  |  | 19 | 17 | 15 | 13 | 11 | 9  |   |   |   |   |
|  |  |  |  |  |  |  |  |    |    |    |    |    | 6  |   |   |   |   |
|  |  |  |  |  |  |  |  | 12 | 11 | 9  | 7  | 5  | 3  | 1 | 4 |   |   |
|  |  |  |  |  |  |  |  | 13 | 14 | 11 | 8  | 5  | 2  | S | 2 |   |   |
|  |  |  |  |  |  |  |  | 19 | 16 | 13 | 16 | 14 | 11 | 8 | 5 | 2 | 5 |
|  |  |  |  |  |  |  |  | 19 | 17 | 19 | 17 | 14 | 11 | 8 | 5 | 8 |   |

*min-cost path found*

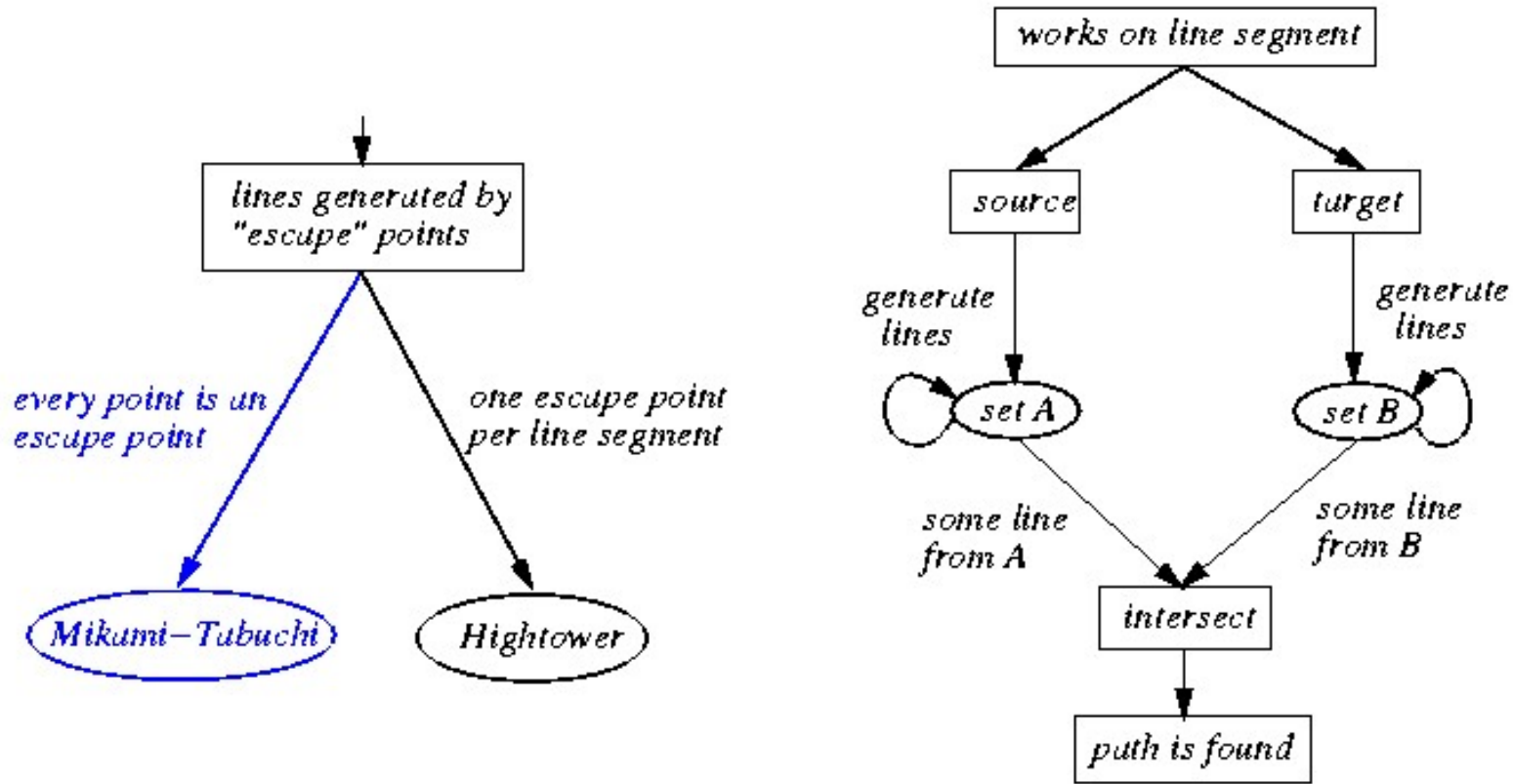
# Line-Search Algorithms

---

- Overcome the drawback of grid representation used by Lee algorithms
  - Time & space complexity for an  $M \times N$  grid:  $O(MN)$
- Consider the base case
  - if no obstacles, two points S and T are to be connected, then a vertical line passing through S and a horizontal line passing through T naturally intersect.



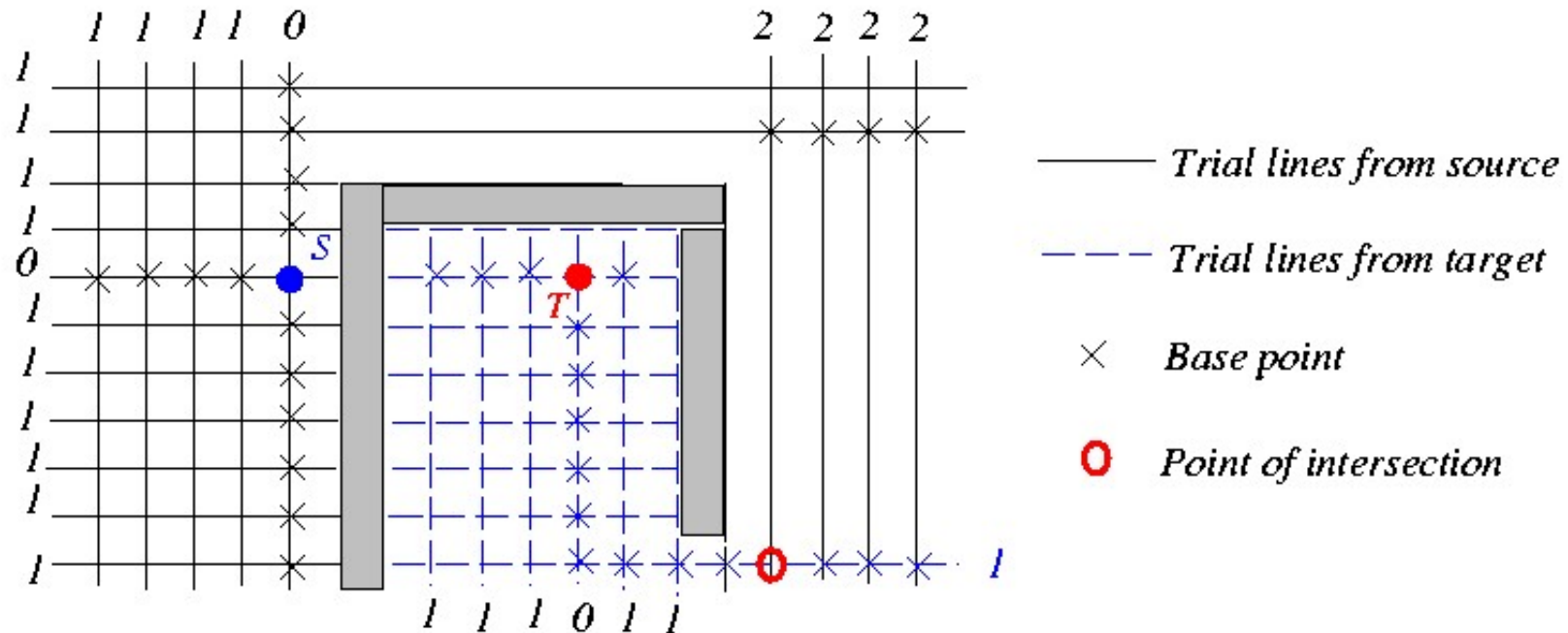
# Features of Line-Search Algorithms



- Time and space complexities:  $O(L)$ , where  $L$  is the # of line segments generated.

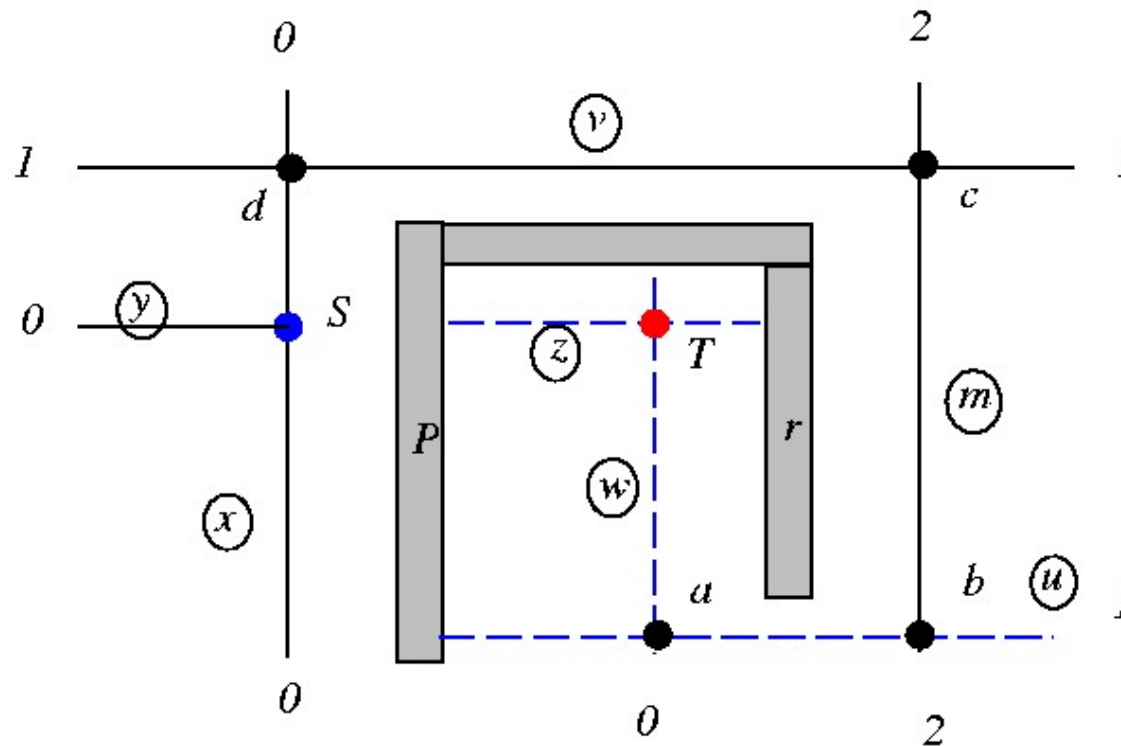
# Mikami-Tabuchi's Algorithm

- Mikami & Tabuchi, "A computer program for optimal routing of printed circuit connectors," *IFIP*, H47, 1968.
- Every grid point is an escape point.



# Hightower's Algorithm

- Hightower, “A solution to line-routing problem on the continuous plane,” DAC-69.
- A single escape point on each line segment.
- If a line parallels to the blocked cells, the escape point is placed just past the endpoint of the segment.





# Comparison of Algorithms

---

|                           | Maze routing |         |         | Line search |           |
|---------------------------|--------------|---------|---------|-------------|-----------|
|                           | Lee          | Soukup  | Hadlock | Mikami      | Hightower |
| Time                      | $O(MN)$      | $O(MN)$ | $O(MN)$ | $O(L)$      | $O(L)$    |
| Space                     | $O(MN)$      | $O(MN)$ | $O(MN)$ | $O(L)$      | $O(L)$    |
| Finds path if one exists? | yes          | yes     | yes     | yes         | no        |
| Is the path shortest?     | yes          | no      | yes     | no          | no        |
| Works on grids or lines?  | grid         | grid    | grid    | line        | line      |

- Soukup, Mikami, and Hightower all adopt some sort of line-search operations  $\Rightarrow$  cannot guarantee shortest paths.

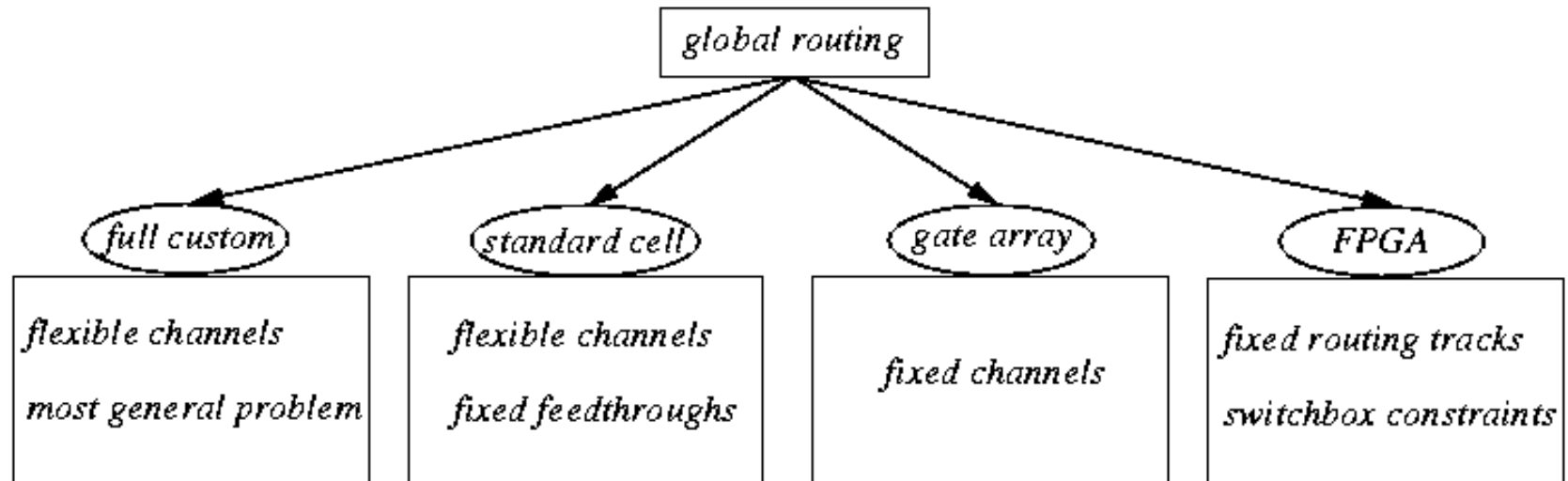
# Global-Routing Problem

---

- Given a netlist  $N = \{N_1, N_2, \dots, N_n\}$ , a routing graph  $G = (V, E)$ , find a Steiner tree  $T_i$  for each net  $N_i$ ,  $1 \leq i \leq n$ , such that  $U(e_j) \leq c(e_j)$ ,  $\forall e_j \in E$  and  $\sum_{i=1}^n L(T_i)$  is minimized, where
  - $c(e_j)$ : capacity of edge  $e_j$ ;
  - $x_{ij} = 1$  if  $e_j$  is in  $T_i$ ;  $x_{ij} = 0$  otherwise;
  - $U(e_j) = \sum_{i=1}^n x_{ij}$ : # of wires that pass through the channel corresponding to edge  $e_j$ ;
  - $L(T_i)$ : total wirelength of Steiner tree  $T_i$ .
- For high-performance, the maximum wirelength ( $\max_{i=1}^n L(T_i)$ ) is minimized (or the longest path between two points in  $T_i$  is minimized).

# Global Routing in different Design Styles

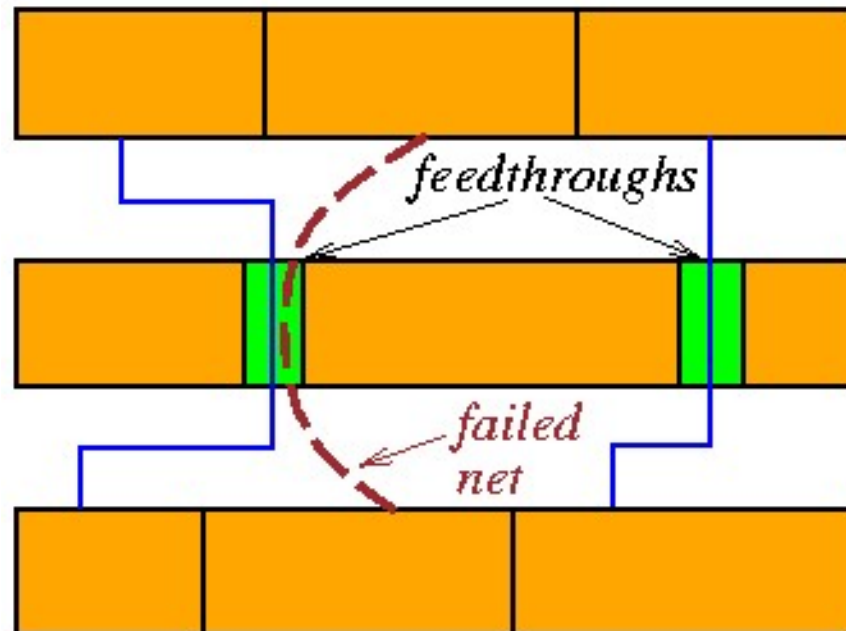
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# Global Routing in Standard Cell

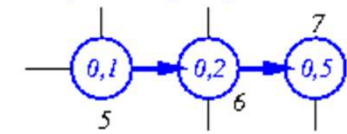
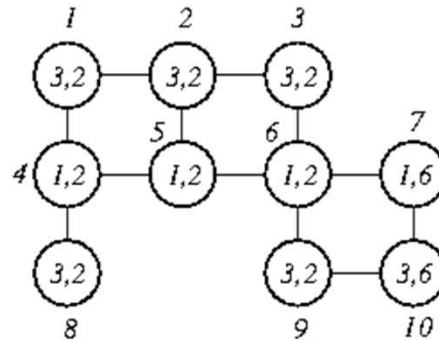
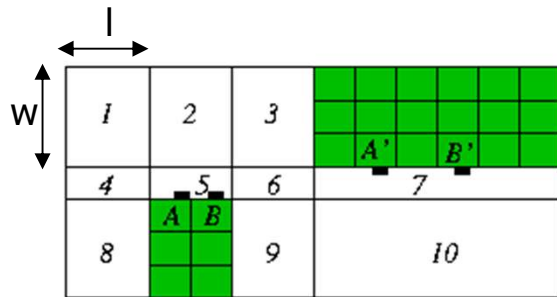
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- Objective
  - Minimize total channel height.
  - Assignment of **feedthrough**: Placement? Global routing?
- For high performance,
  - Minimize the maximum wire length.
  - Minimize the maximum path length.

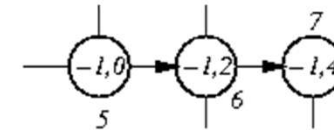


# Global-Routing: Maze Routing

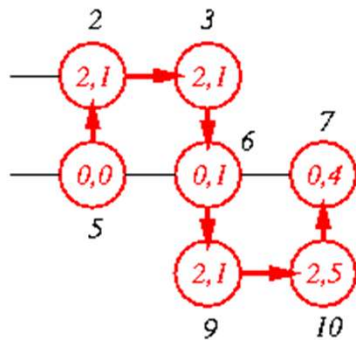
- Routing channels may be modeled by a weighted undirected graph called **channel connectivity graph**.
  - Node  $\leftrightarrow$  channel; edge  $\leftrightarrow$  two adjacent channels; capacity: (width, length)



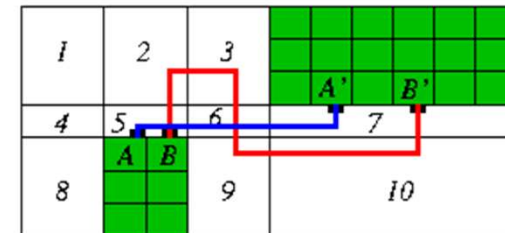
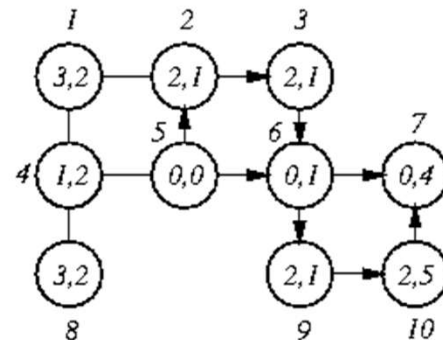
route A-A' via 5-6-7



route B-B' via 5-6-7



route B-B' via 5-2-3-6-9-10-7 updated channel graph



maze routing for nets A and B

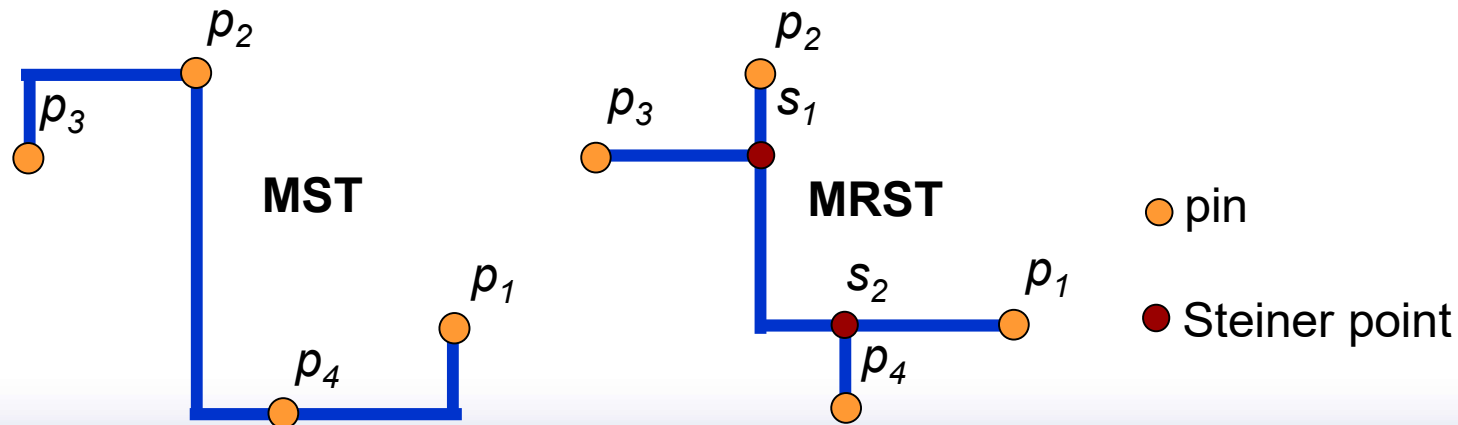
- Update:
  - If a path bends in a channel, both width and length are reduced
  - If a path goes only vertically/horizontally, only length/width is reduced

# Routing Tree

- ❑ If all nets are two-pin ones, we can apply a general-purpose routing algorithm to handle the problem, such as maze, line-search, and A\*-search routing.
- ❑ For three or more multi-pin nets, one approach is to *decompose* each net into a set of two-pin connections, and then routes the connections one-by-one.

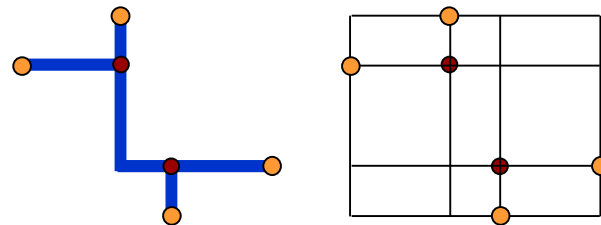
# The Routing-Tree Problem

- ❑ **Problem:** Given a set of pins of a net, interconnect the pins by a “routing tree.”
- ❑ **Minimum Spanning Tree (MST):** a minimum-length tree of edges connecting all the pins
- ❑ **Minimum Rectilinear Steiner Tree (MRST) Problem:** Given  $n$  points in the plane, find a minimum-length tree of rectilinear edges which connects the points.
- ❑  $MRST(P) = MST(P \cup S)$ , where  $P$  and  $S$  are the sets of original points and Steiner points, respectively.



## Theoretic Results for the MRST Problem

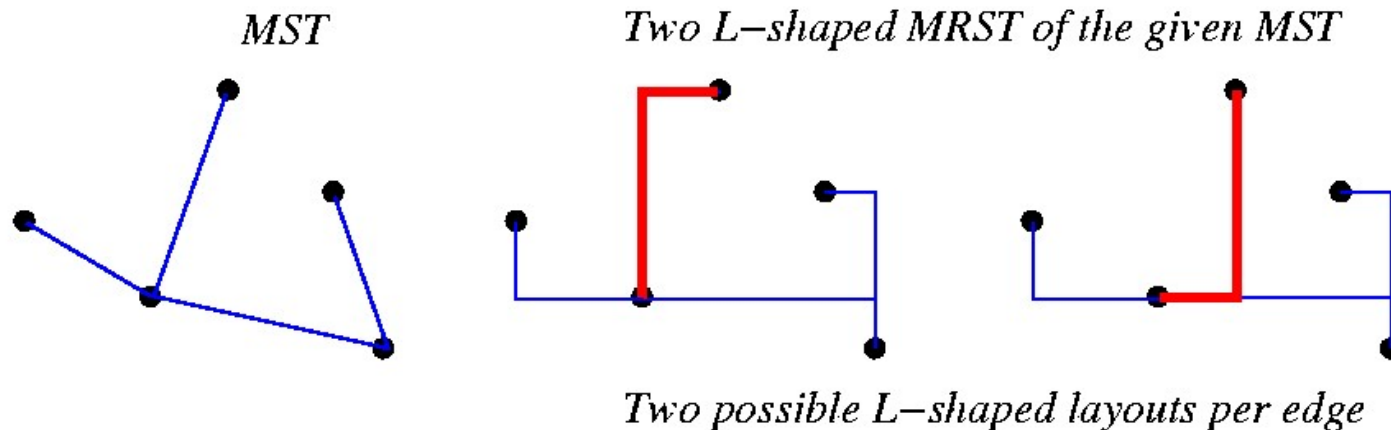
- **Hanan's Thm:** There exists an MRST with all Steiner points (set  $S$ ) chosen from the intersection points of horizontal and vertical lines drawn through points of  $P$ .
  - Hanan, "On Steiner's problem with rectilinear distance," *SIAM J. Applied Math.*, 1966.



- **Hwang's Theorem:** For any point set  $P$ ,  $\frac{Cost(MST(P))}{Cost(MRST(P))} \leq \frac{3}{2}$ .
  - Hwang, "On Steiner minimal tree with rectilinear distance," *SIAM J. Applied Math.*, 1976.
- Better approximation algorithm with the performance bound  $61/48$ 
  - Foessmeier *et al*, "Fast approximation algorithm for the rectilinear Steiner problem," Wilhelm Schickard-Institut für Informatik, TR WSI-93-14, 93.

# Coping with the MRST Problem

- Ho, Vijayan, and Wong, “New algorithms for the rectilinear Steiner problem,” TCAD-90.
  1. Construct an MRST from an MST.
  2. Each edge is straight or L-shaped.
  3. Maximize overlaps by dynamic programming.
- About 8% smaller than  $Cost(MST)$ .



# Iterated 1-Steiner Heuristic for MRST

- Kahng & Robins, “A new class of Steiner tree heuristics with good performance: the iterated 1-Steiner approach,” *ICCAD-90*.

## Algorithm: Iterated\_1-Steiner( $P$ )

$P$ : set  $P$  of  $n$  points.

1 **begin**

2  $S \leftarrow \emptyset$ ;

/\*  $H(P \cup S)$ : set of Hanan points \*/

/\*  $\Delta MST(A, B) = Cost(MST(A)) - Cost(MST(A \cup B))$  \*/

3 **while** ( $Cand \leftarrow \{x \in H(P \cup S) \mid \Delta MST(P \cup S, \{x\}) > 0\} \neq \emptyset$ ) **do**

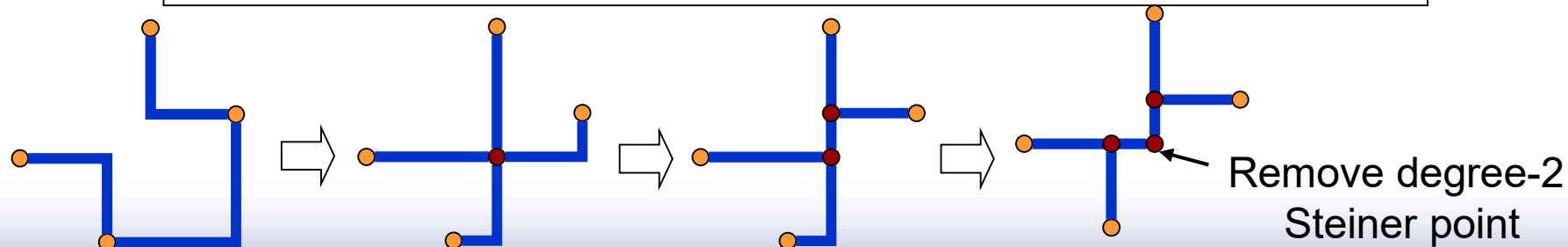
4 Find  $x \in Cand$  and which maximizes  $\Delta MST(P \cup S, \{x\})$ ;

5  $S \leftarrow S \cup \{x\}$ ;

6 Remove points in  $S$  which have degree  $\leq 2$  in  $MST(P \cup S)$ ;

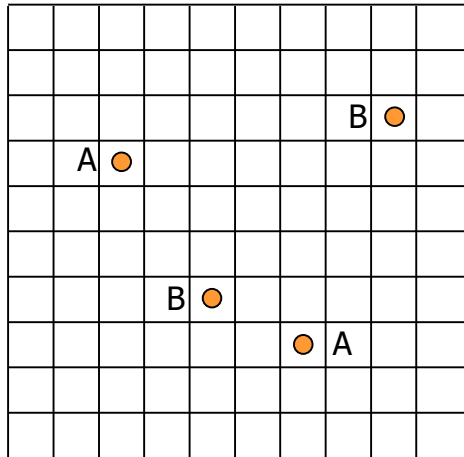
7 **Output**  $MST(P \cup S)$ ;

8 **end**

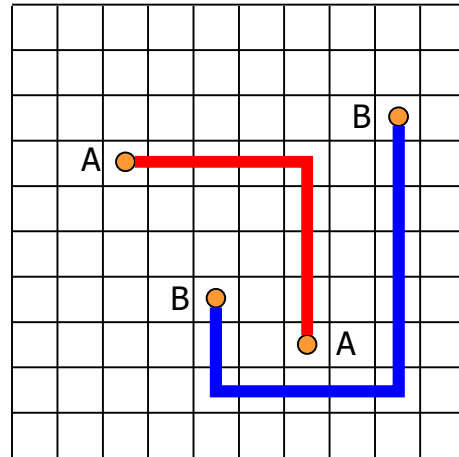


# Net Ordering

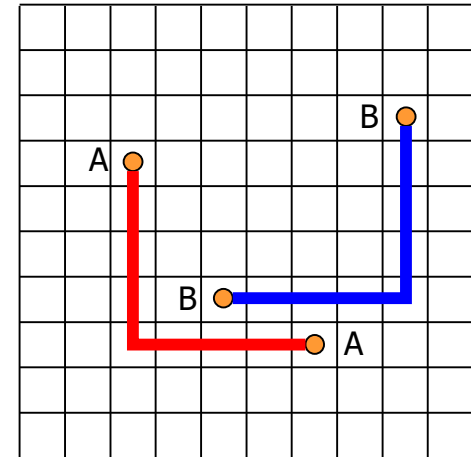
- Net ordering greatly affects routing solutions
- Finding the optimal net ordering is proven to be NP-hard
  - Abel, “On the ordering of connections for automatic wire routing,” TC-1972.
- In the example, we should route net *b* before net *a*



**A one-layer routing instance with nets *A* and *B***



**Route *A* before *B***



**Route *B* before *A***

# *Net Ordering (cont'd)*

- ❑ Order the nets in the ascending order of the # of pins within their bounding boxes.
- ❑ Order the nets in the ascending (descending) order of their lengths if routability (timing) is the most critical metric
- ❑ Order the nets based on their timing criticality.

# *Rip-Up and Re-routing*

- ❑ Rip-up and re-routing is required if a global or detailed router fails in routing all nets.
- ❑ Approaches: the manual approach? the automatic procedure?
- ❑ Two steps in rip-up and re-routing
  1. Identify bottleneck regions, rip off some already routed nets.
  2. Route the blocked connections, and re-route the ripped-up connections.
- ❑ Repeat the above steps until all connections are routed or a time limit is exceeded.