

Logic design (2016 spring)

Quiz # 4

Name: _____ ID: _____

1. A combinational circuit has four binary inputs (A, B, C, D) and three binary outputs (X, Y, Z). XYZ represents a binary number whose value equals the number of 0's at the input.

For example, if ABCD = 1011 then XYZ = 001.

(a) (15%) Complete the true table for the circuit.

(b) (15%) Find the minterm expansions for X using *m*-notation.

(c) (16%) Find the maxterm expansions for Z using *M*-notation.

Note that the order of literals for *m*-notation and *M*-notation is ABCD.

ABCD	X	Y	Z
0000	1	0	0
0001	0	1	1
0010	0	1	1
0011	0	1	0
0100	0	1	1
0101	0	1	0
0110	0	1	0
0111	0	0	1
1000	0	1	1
1001	0	1	0
1010	0	1	0
1011	0	0	1
1100	0	1	0
1101	0	0	1
1110	0	0	1
1111	0	0	0

$$X = m(0) = A'B'C'D'$$

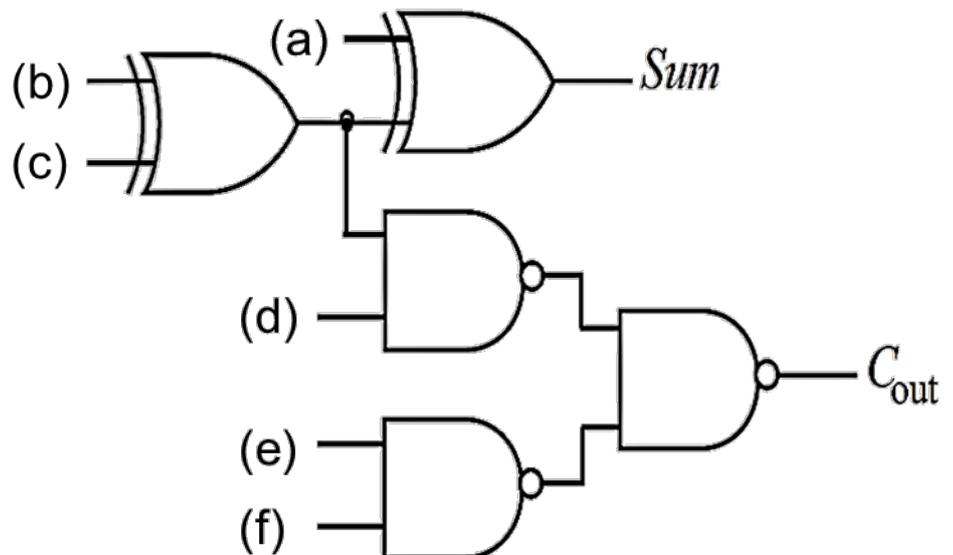
$$Z = \prod M(0, 3, 5, 6, 9, 10, 12, 15)$$

$$= (A' + B' + C' + D')(A' + B' + C + D)(A' + B + C' + D)(A' + B + C + D')(A + B' + C' + D)(A + B' + C + D')(A + B + C' + D)(A + B + C + D)$$

2. (54%) A full adder has three binary inputs (X, Y, C_{in}) and two binary outputs (C_{out}, Sum).
The true table for the full adder is given below. Please complete the logic circuit of the full adder by filling in the blanks for (a) ~ (f) with X, Y , or C_{in} .

X	Y	C_{in}	C_{out}	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

(a)	C_{in}	X	Y
(b)	X	C_{in}	X
(c)	Y	Y	C_{in}
(d)	C_{in}	X	Y
(e)	X	C_{in}	X
(f)	Y	Y	C_{in}



$$\begin{aligned} Sum &= X'Y'C_{in} + X'YC_{in}' + XY'C_{in}' + XYC_{in} = (X'Y' + XY)C_{in} + (X'Y + XY')C_{in}' \\ &= (X \oplus Y)'C_{in} + (X \oplus Y)C_{in}' = X \oplus Y \oplus C_{in} = (X \oplus Y) \oplus C_{in} \end{aligned}$$

$$\begin{aligned} C_{out} &= X'YC_{in} + XY'C_{in} + XYC_{in}' + XYC_{in} = (X'Y + XY')C_{in} + XY(C_{in}' + C_{in}) = (X \oplus Y)C_{in} + XY \\ &= (((X \oplus Y)C_{in} + XY)')' = (((X \oplus Y)C_{in})'(XY)')' \end{aligned}$$