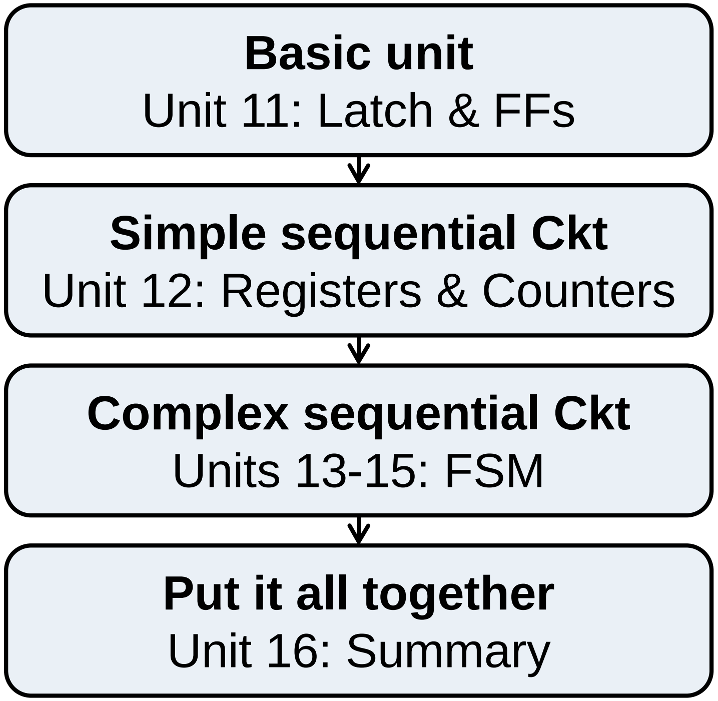
# UNIT 12
# REGISTERS AND COUNTERS

Spring 2011

# Registers and Counters

- **Contents**
  - Register and register transfers
    - Operation
    - Shift register
  - Counters
    - Design of binary counters
    - Counters for other sequences
    - Counter design using D FFs
    - Counter design using S-R and J-K
    - Derivation of FF input equations
- **Reading**
  - Unit 12



**Basic unit**
Unit 11: Latch & FFs

**Simple sequential Ckt**
Unit 12: Registers & Counters

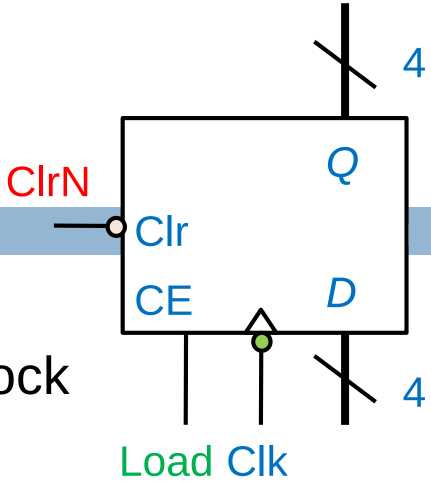
**Complex sequential Ckt**
Units 13-15: FSM

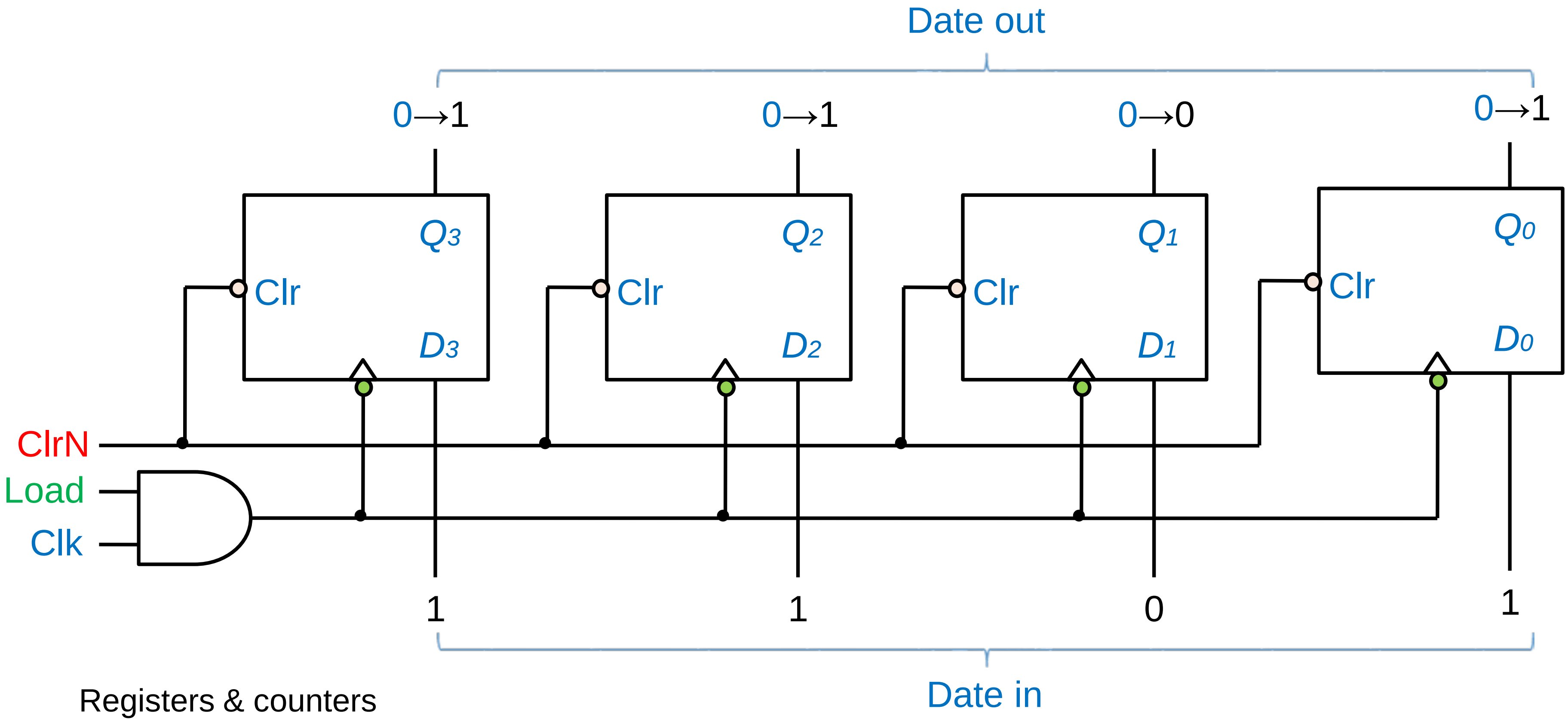
**Put it all together**
Unit 16: Summary

# Registers (1/2)

- **A register: a group of D FFs with a common clock**
  - e.g., 4-bit D FF registers with Data, Load, Clear, Clock
    1. Using gated clock

       When Load = 1, load data at *D* to *Q* at Clk falling



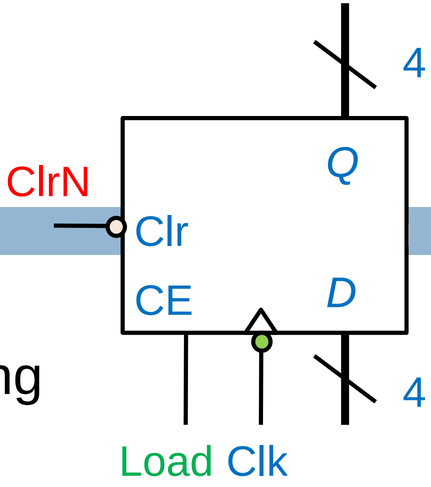


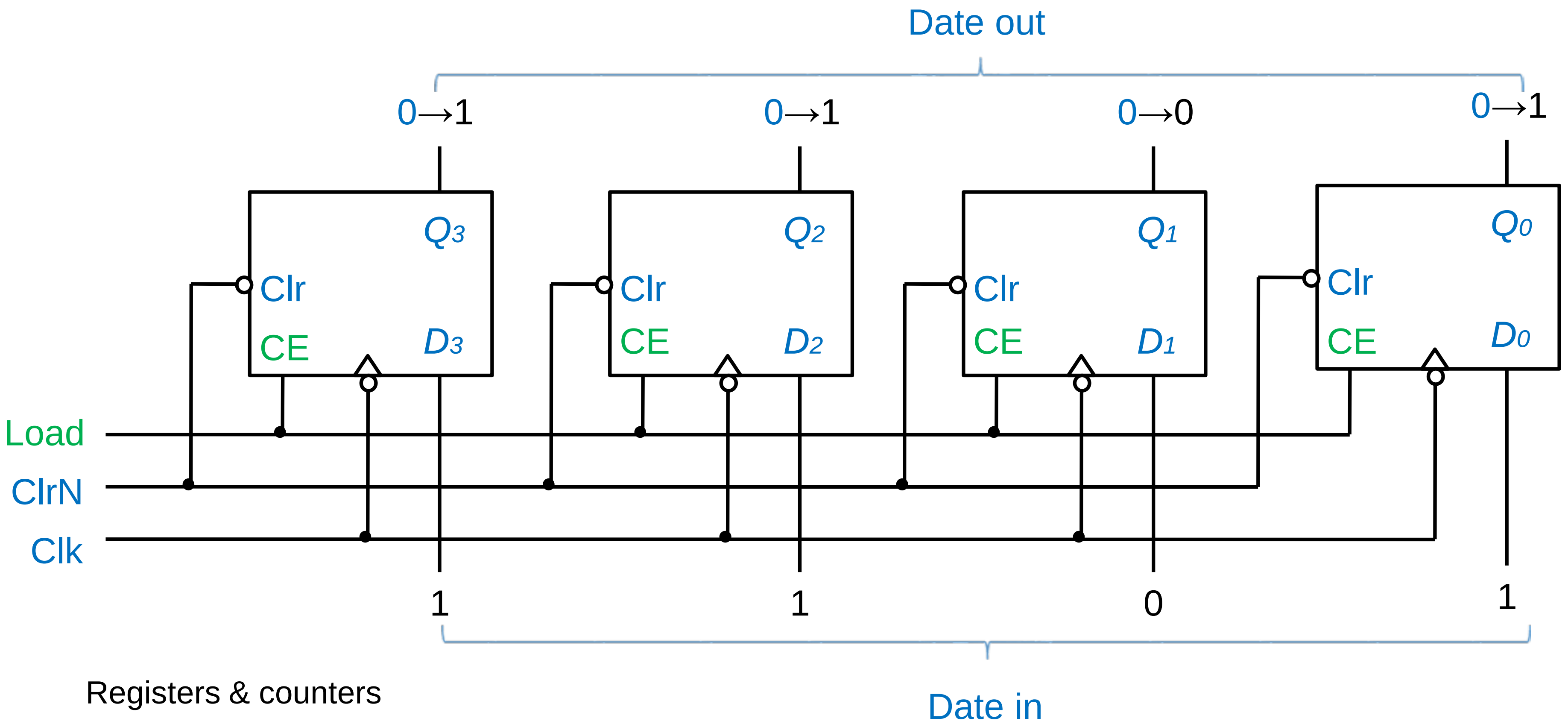
# Registers (2/2)

2. With clock enable

   When Load = 1, load data at *D* to *Q* at Clk falling

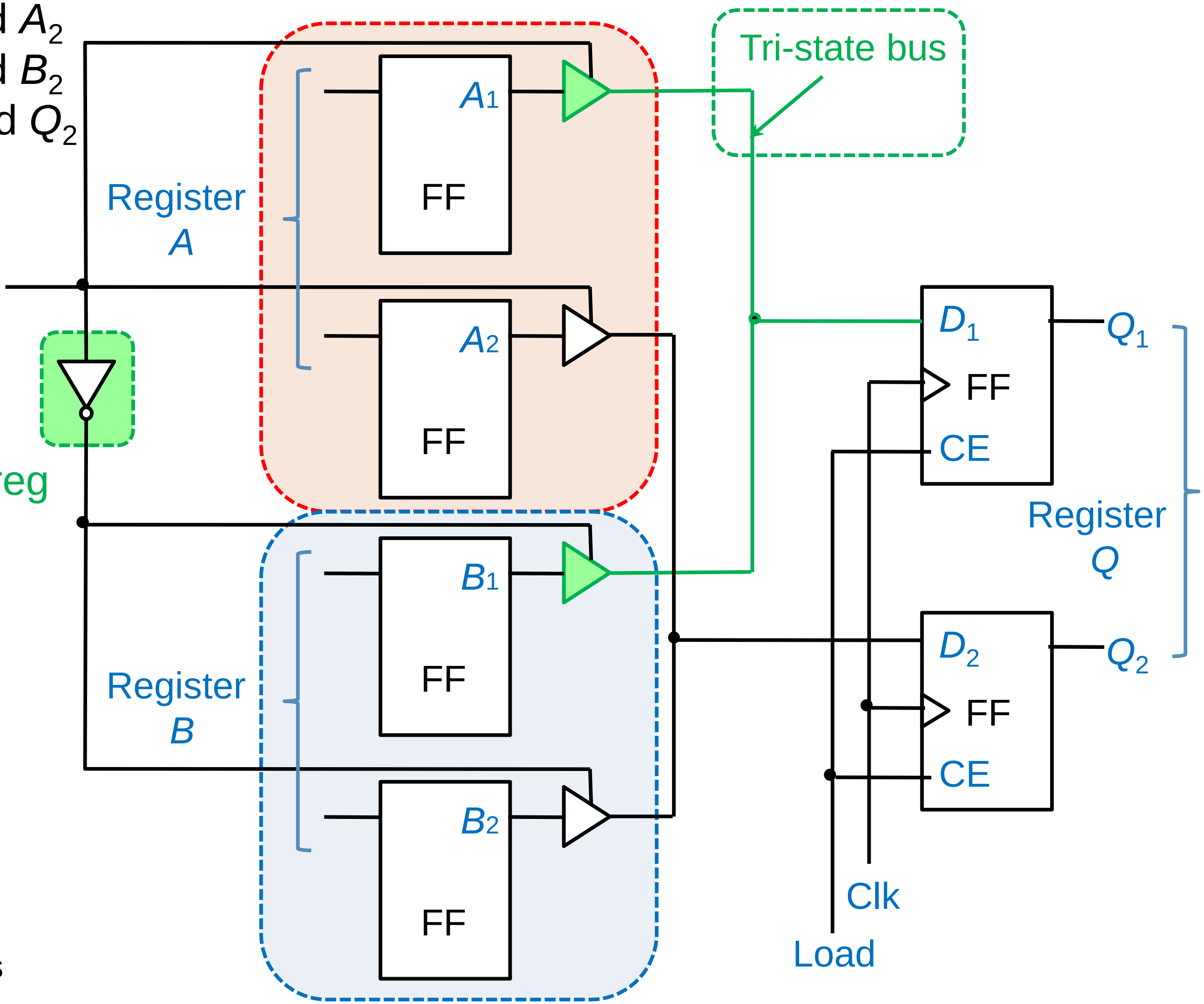
   When ClrN = 0, clear *Q* to 0





# Data Transfer between Registers

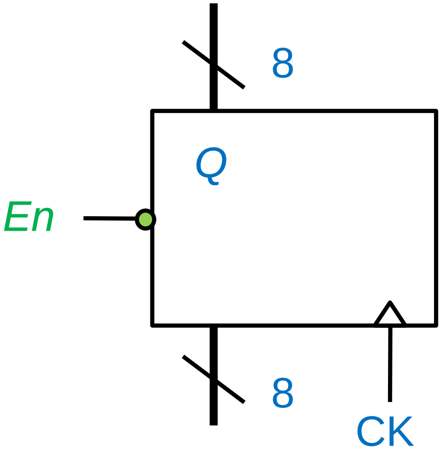
Reg $A$ = FFs $A_1$ and $A_2$
Reg $B$ = FFs $B_1$ and $B_2$
Reg $Q$ = FFs $Q_1$ and $Q_2$



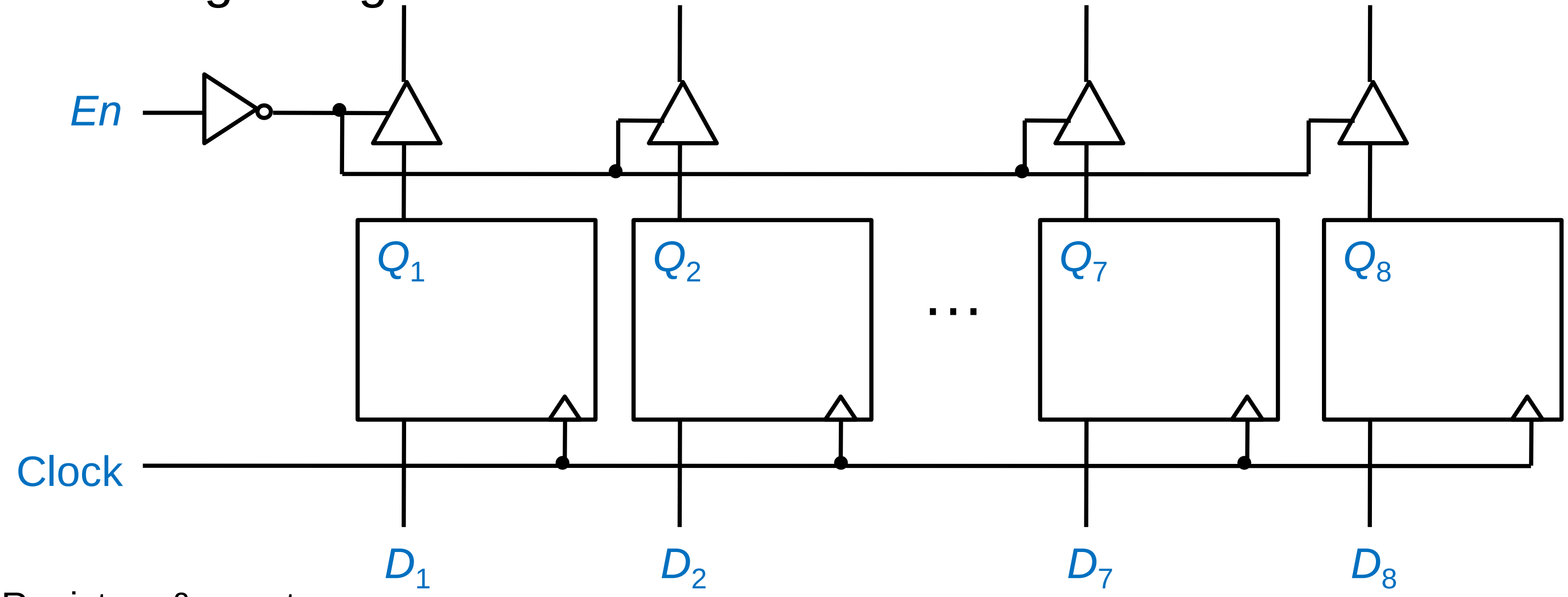
Data transfer btw reg
≡ 2:1 MUX
If $En$ = 1, $Q \Leftarrow A$;
else, $Q \Leftarrow B$

# 8-Bit Register with Tri-State Output (1/2)

- Symbol

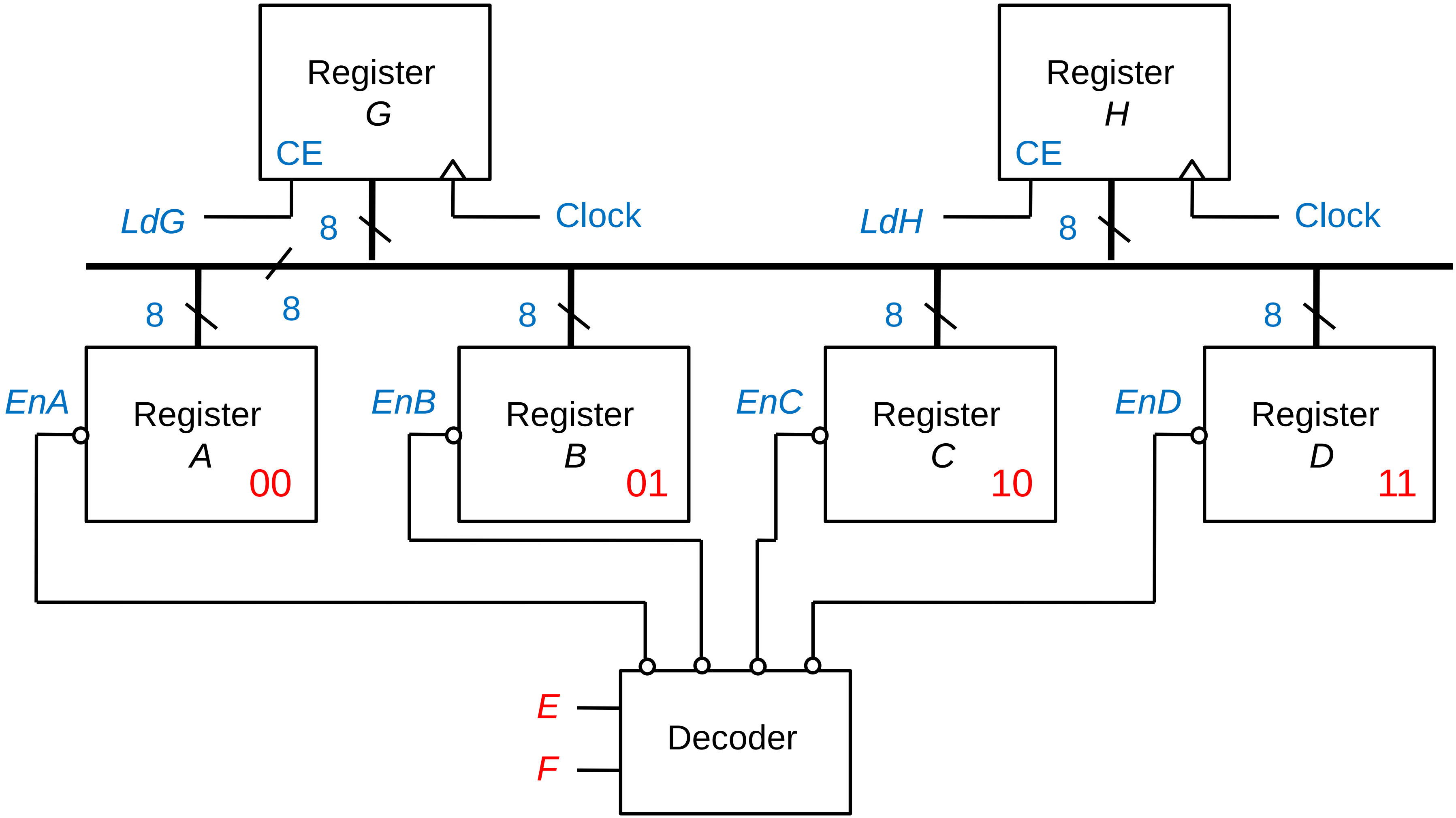


- Logic diagram



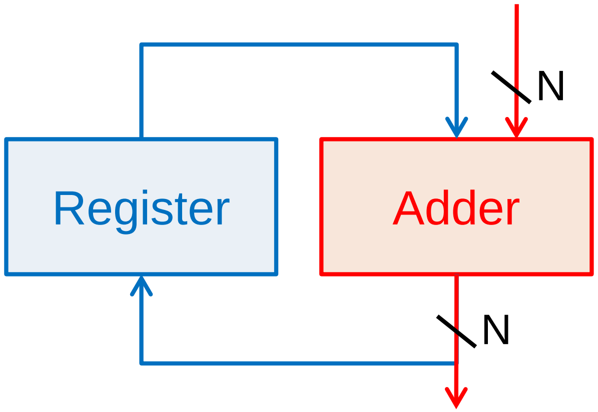
# 8-Bit Register with Tri-State Output (2/2)

- Data transfer

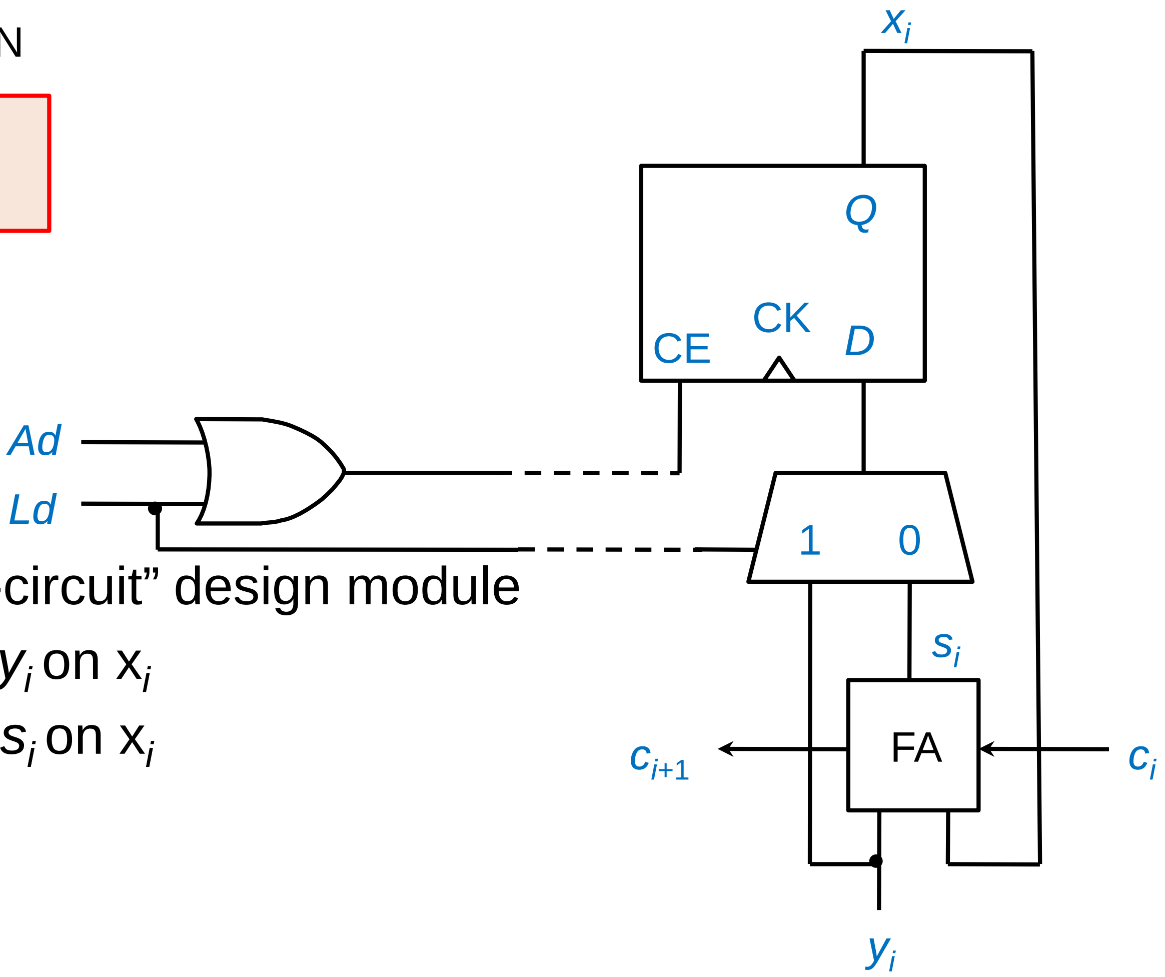


# N-Bit Parallel Adder with Accumulator (1/2)

- **Adder with accumulater**

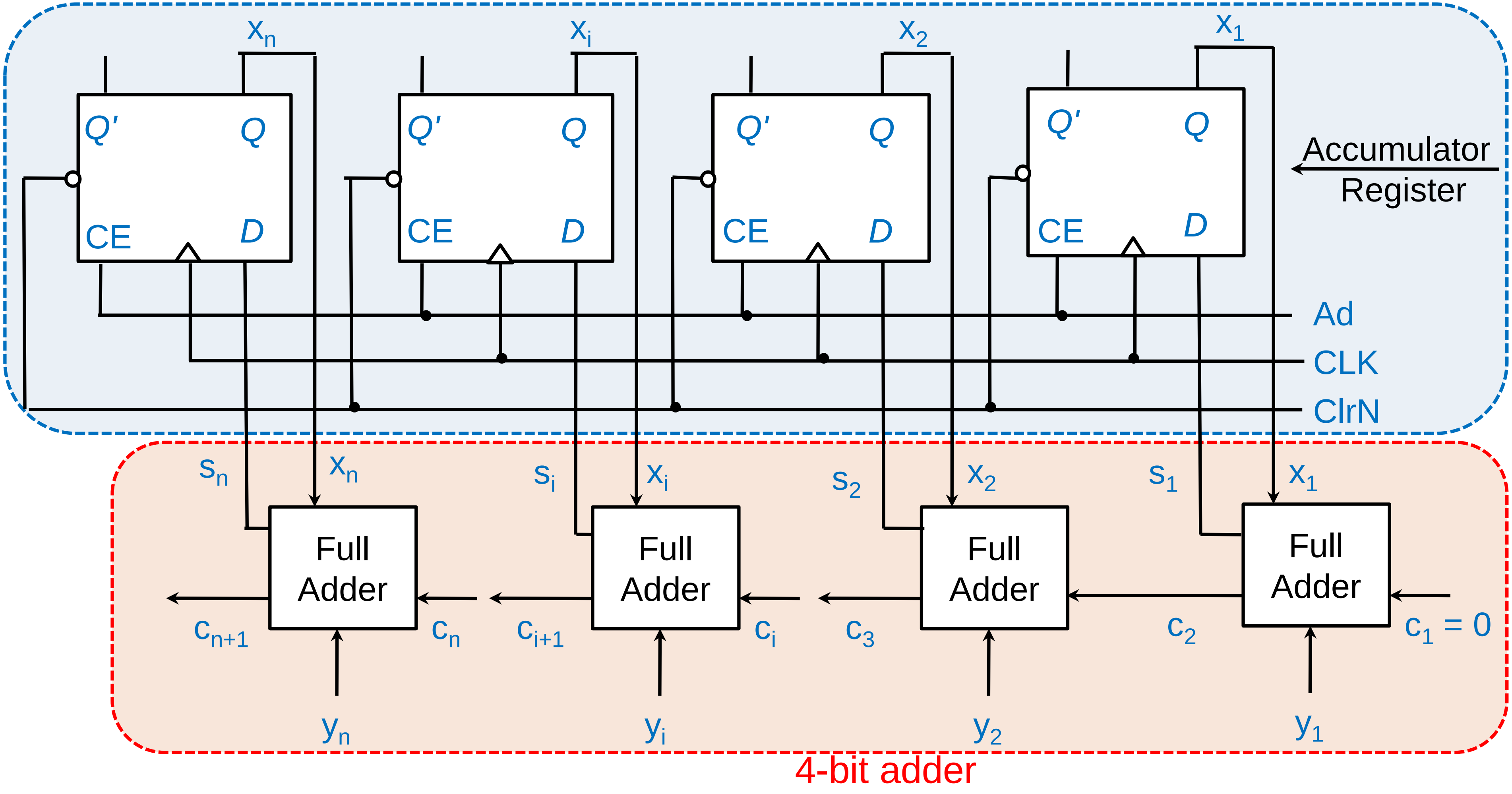


- **Adder with MUX**
  - Modular design
  - Suitable for “sub-circuit” design module
    1. Set Ld = 1; put $y_i$ on x$_i$
    2. Set Ad = 1; put $s_i$ on x$_i$



# N-Bit Parallel Adder with Accumulator (2/2)

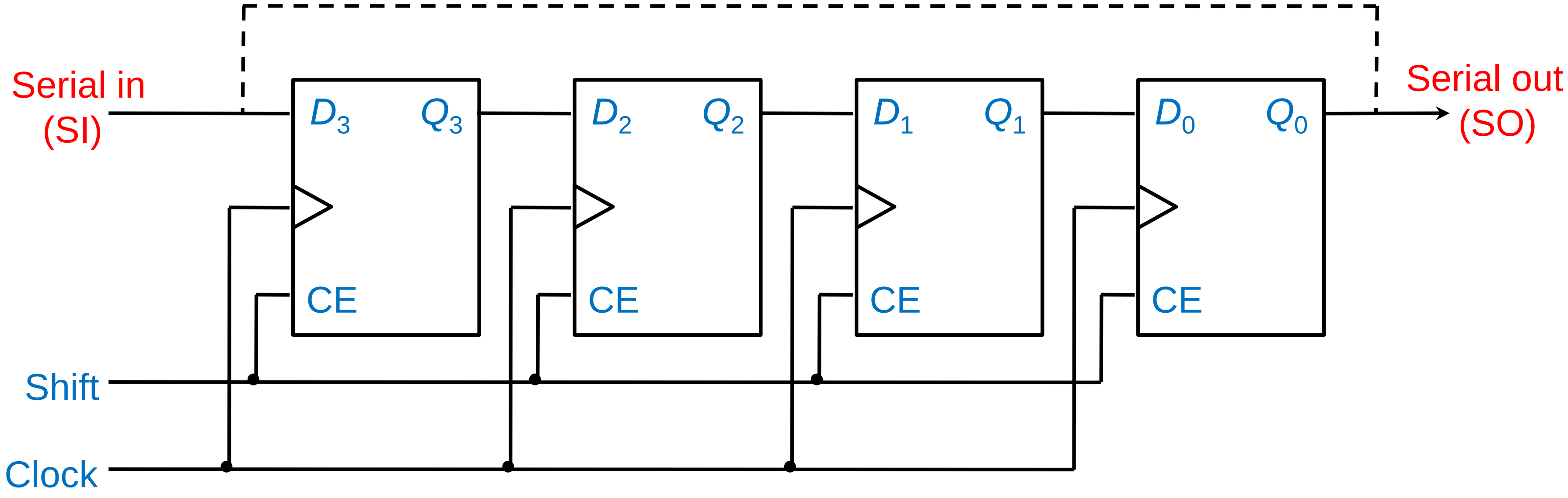
- **Down to bit-level**



# Shift Registers

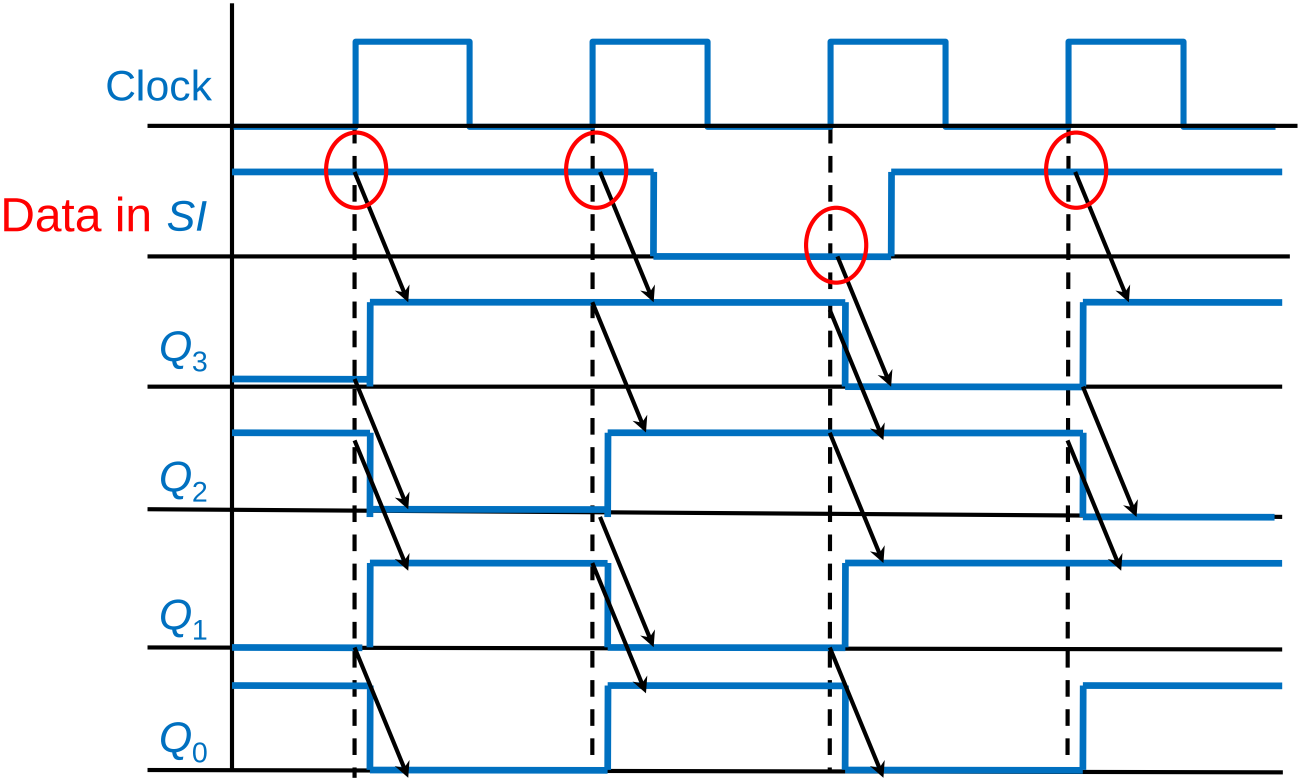
# Shift Registers (1/2)

- **A shift register: a group of FFs where binary data can be stored and shifted left or right when a shift signal is applied**
  - e.g., 4-bit right-shift register



# Shift Registers (2/2)

- Timing diagram of a 4-bit right-shift register



Initial, $Q_3Q_2Q_1Q_0$=0101

*SI* = 1, 1, 0, 1

⇒ $Q_3Q_2Q_1Q_0$

0101

1010

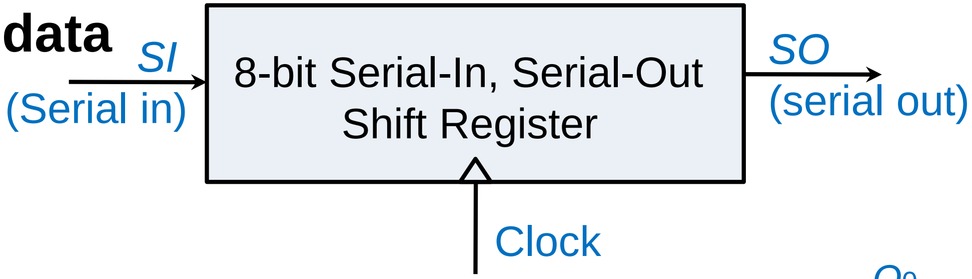
1101

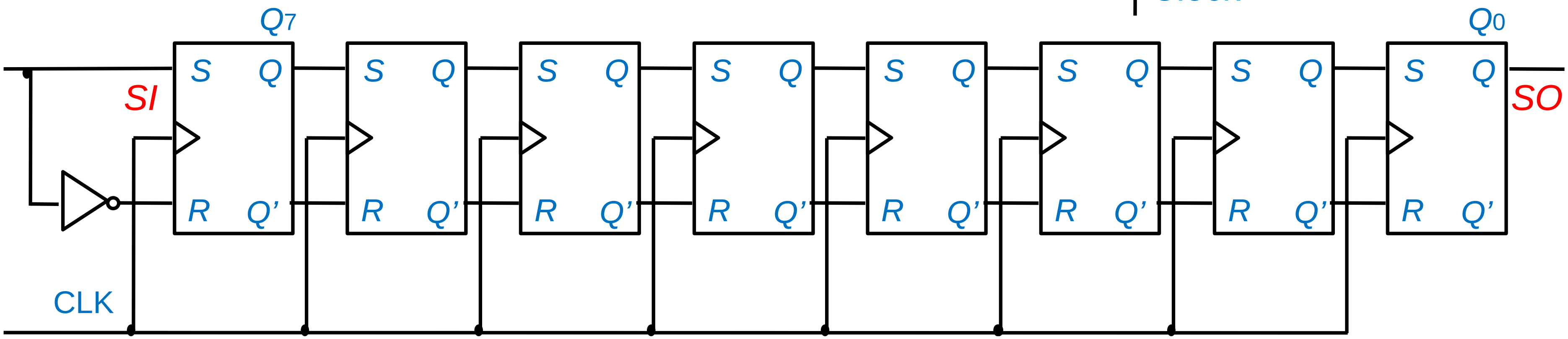
0110

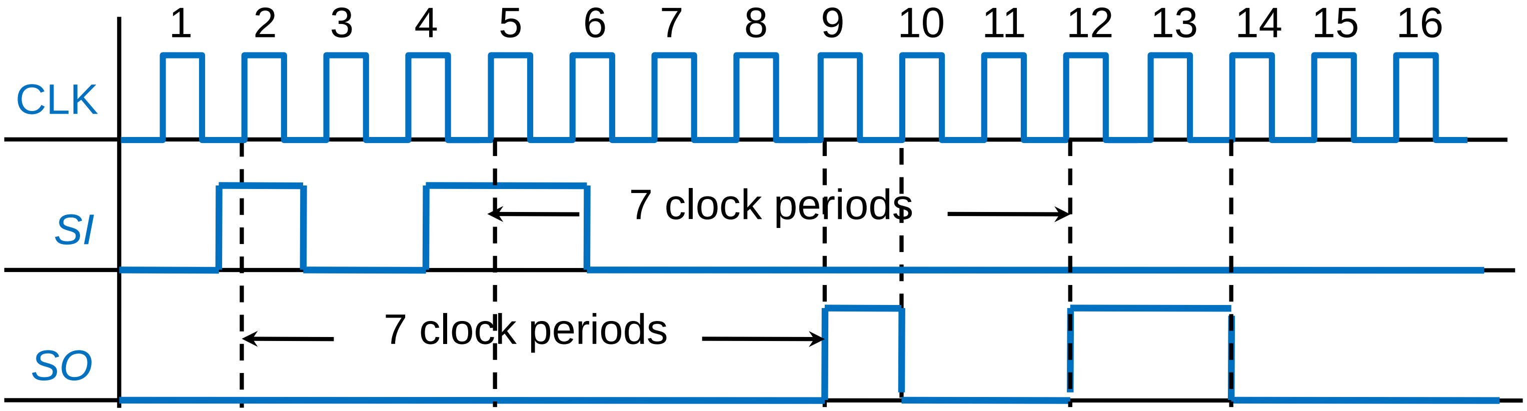
1011

# N-bit Serial-In Serial-Out Shift Registers

- **Take (*n*-1) cycles to output data**





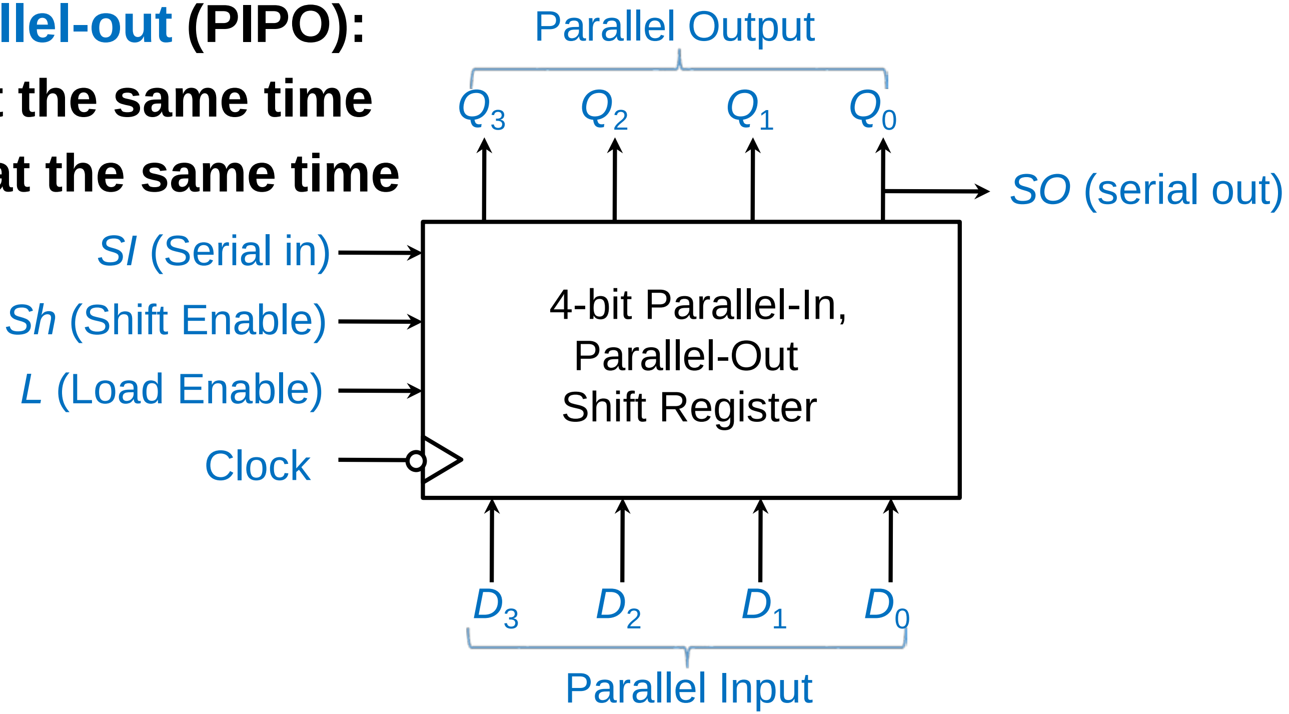


# Parallel-In Parallel-Out Right Shift Register

- **Parallel-in parallel-out (PIPO):**

  **Load all data at the same time**

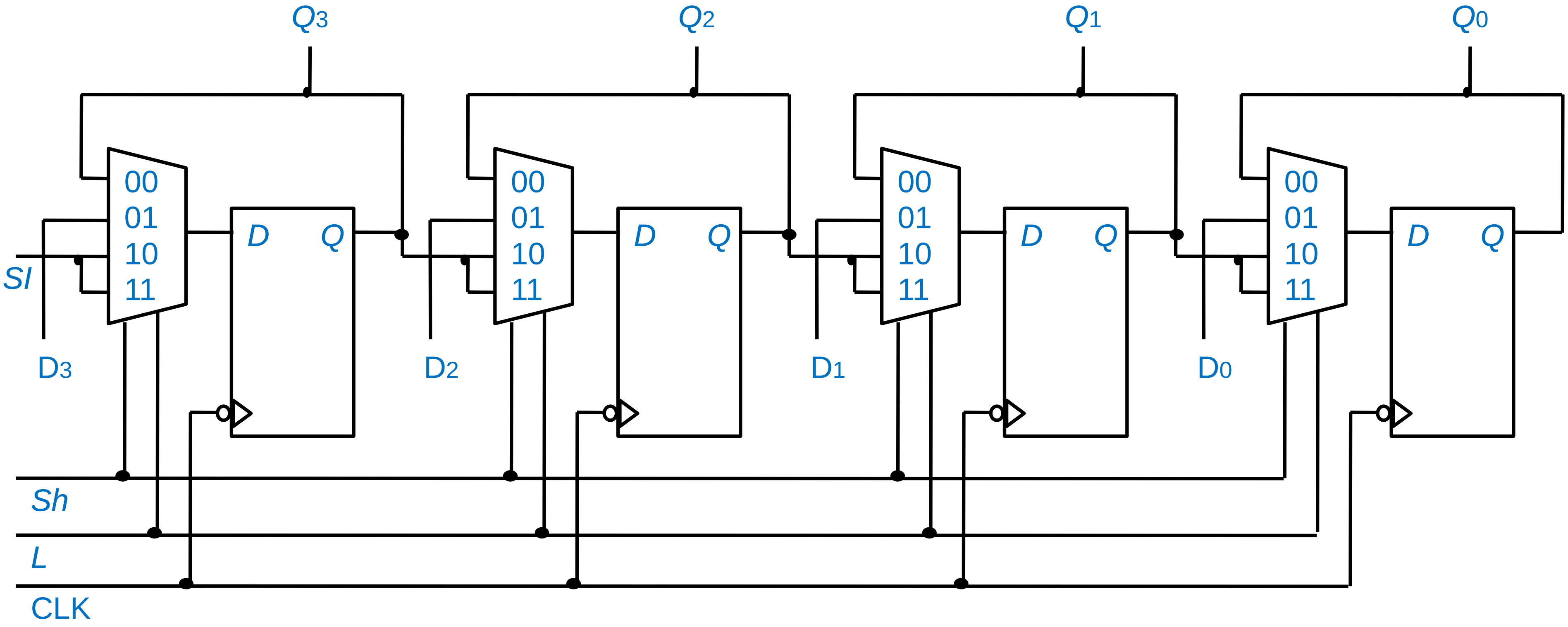
  **Read out data at the same time**



| Inputs | | Next state | | | | Action |
|---|---|---|---|---|---|---|
| $Sh$(Shift) | $L$(Load) | $Q_3^+$ | $Q_2^+$ | $Q_1^+$ | $Q_0^+$ | |
| 0 | 0 | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ | no change |
| 0 | 1 | $D_3$ | $D_2$ | $D_1$ | $D_0$ | load |
| 1 | X | $SI$ | $Q_3$ | $Q_2$ | $Q_1$ | Right shift |

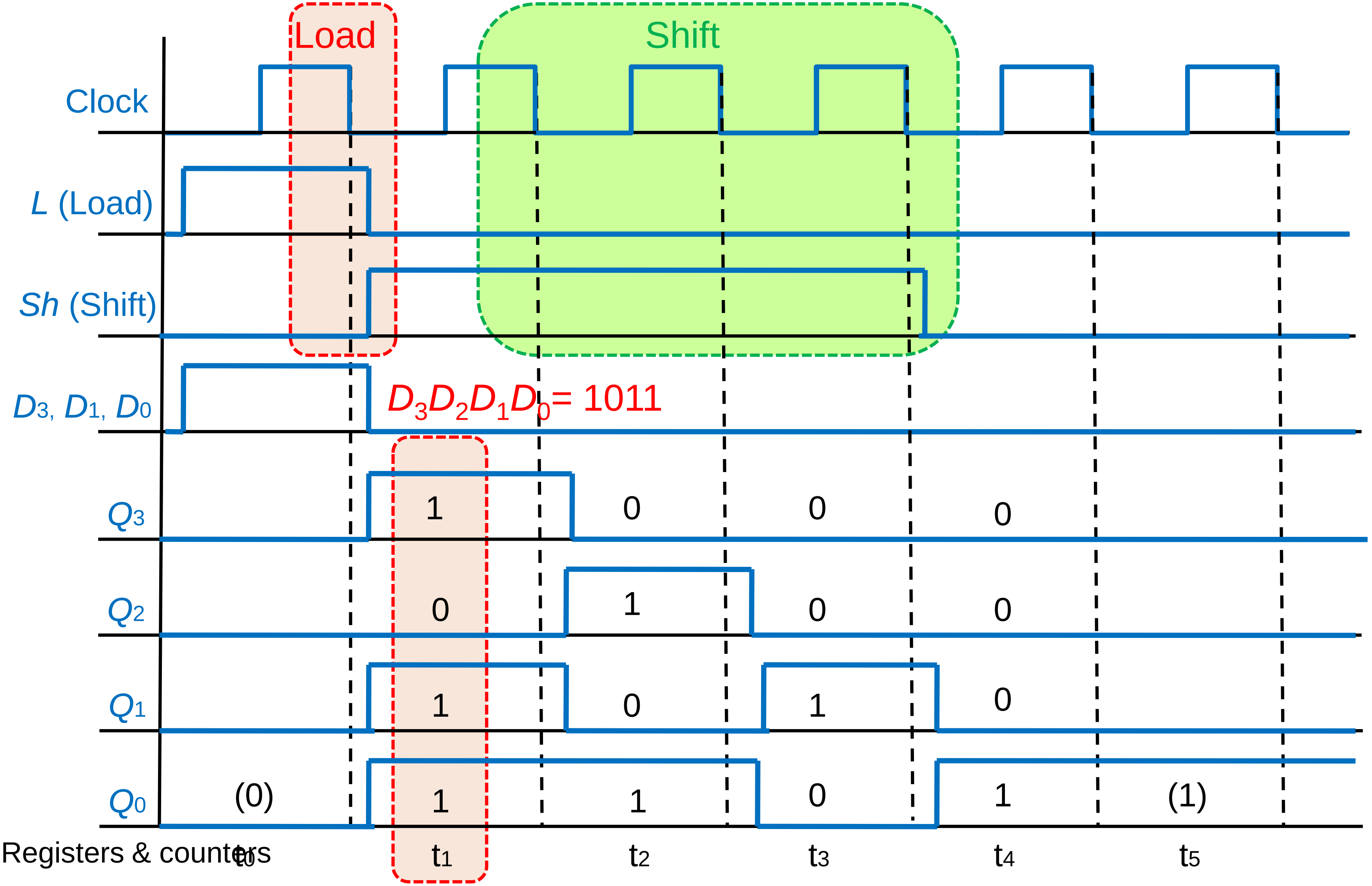
# PIPO Shift Register Implementation (1/2)

- **Implement using FFs and MUXes**



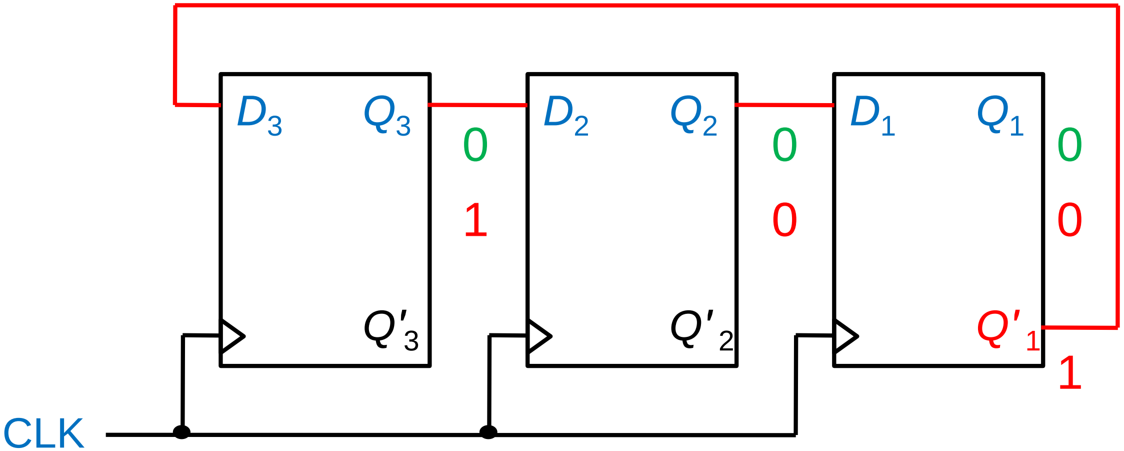
# PIPO (2/2)

| *Sh*(Shift) | *L*(Load) | $Q_3^+$ | $Q_2^+$ | $Q_1^+$ | $Q_0^+$ | Action |
|---|---|---|---|---|---|---|
| 0 | 0 | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ | no change |
| 0 | 1 | $D_3$ | $D_2$ | $D_1$ | $D_0$ | load |
| 1 | X | *SI* | $Q_3$ | $Q_2$ | $Q_1$ | Right shift |

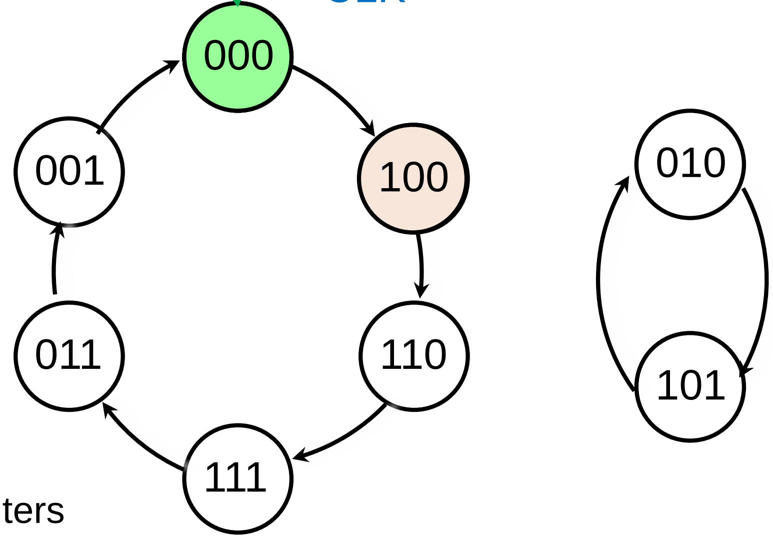


# Shift Register with Inverted Feedback

- **Johnson counter: a shift register with inverted feedback**
  - Counter: a circuit that cycles through a fixed sequence of states



- **State graph**



# Binary Counters

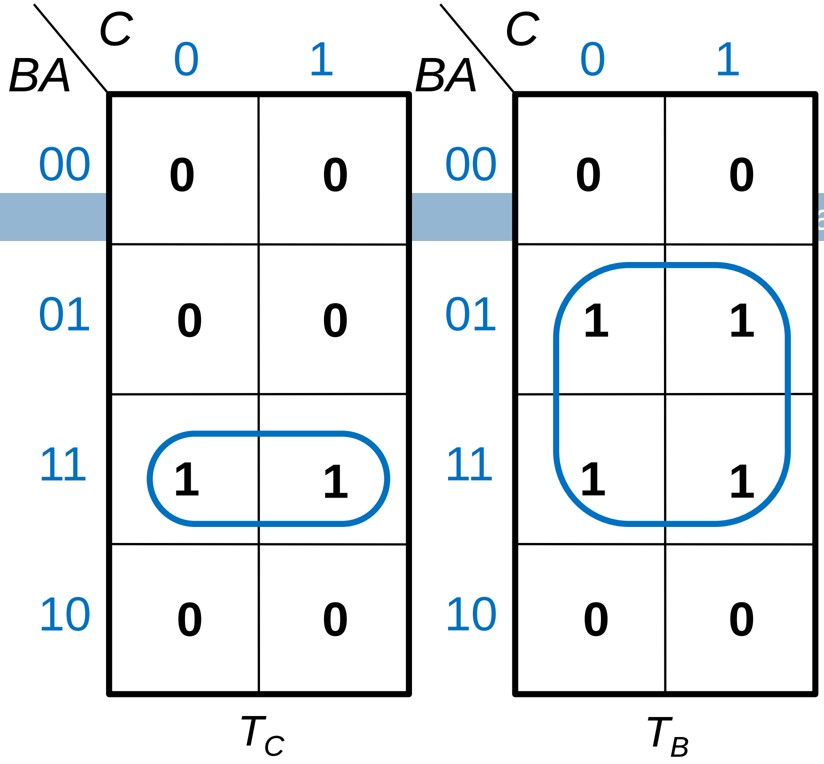
## Synchronous counters discussed

# Counting 0—7 (1/3)

- A **synchronous counter**: FFs are synchronized by a common clock
  - e.g., count 0~7

1. T FF?

| Present State |  |  | Next State |  |  | FF Inputs |  |  |
|---|---|---|---|---|---|---|---|---|
| $C$ | $B$ | $A$ | $C^+$ | $B^+$ | $A^+$ | $T_C$ | $T_B$ | $T_A$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

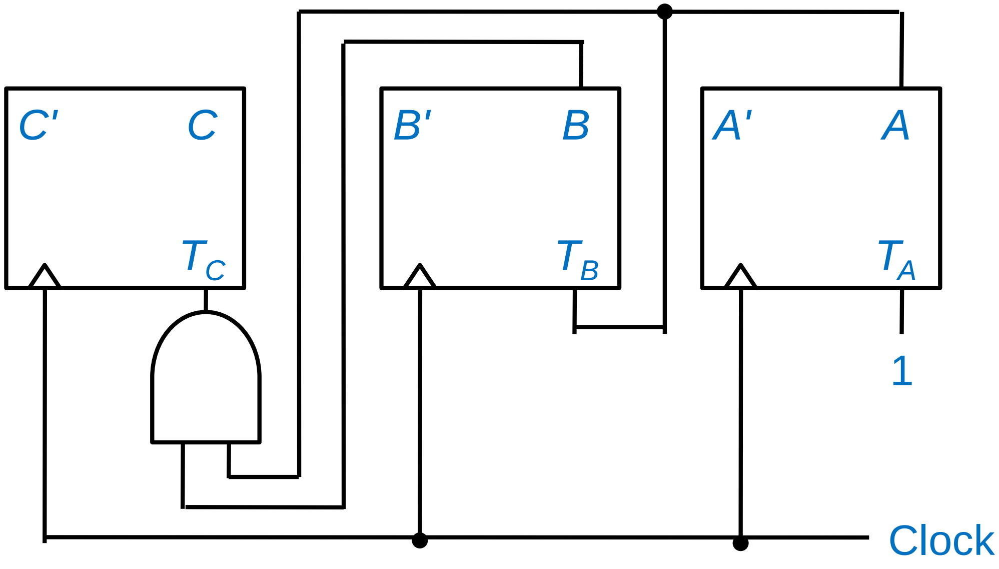


| $BA$ \ $C$ | 0 | 1 |
|---|---|---|
| 00 | 0 | 0 |
| 01 | 0 | 0 |
| 11 | 1 | 1 |
| 10 | 0 | 0 |

$T_C$

| $BA$ \ $C$ | 0 | 1 |
|---|---|---|
| 00 | 0 | 0 |
| 01 | 1 | 1 |
| 11 | 1 | 1 |
| 10 | 0 | 0 |

$T_B$



# Counting 0—7 (2/3)

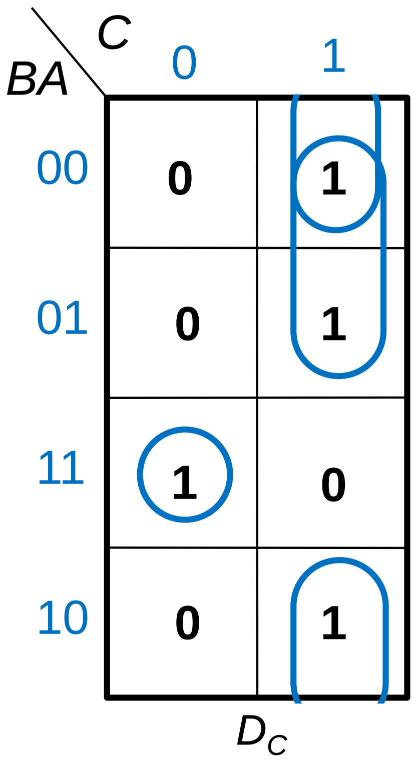
| Present State | | | Next State | | |
|---|---|---|---|---|---|
| $C$ | $B$ | $A$ | $C^+$ | $B^+$ | $A^+$ |
| 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 |

2. D FF?

$D_A = A^+ = A'$

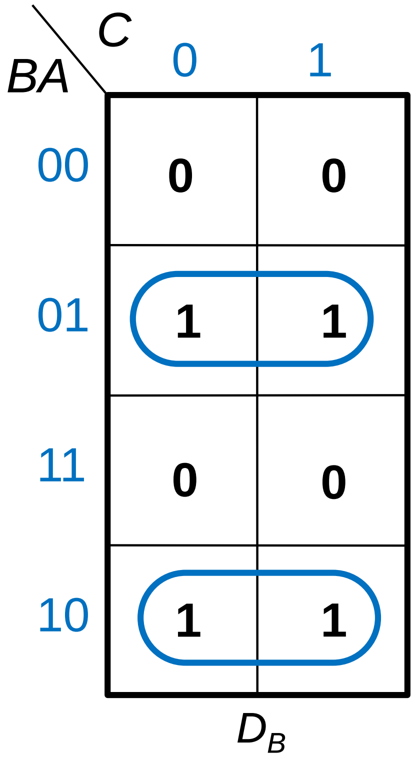
$D_B = B^+ = BA' + B'A = B \oplus A$

$D_C = C^+ = C'BA + CB' + CA' = C'BA + C(BA)' = C \oplus BA$



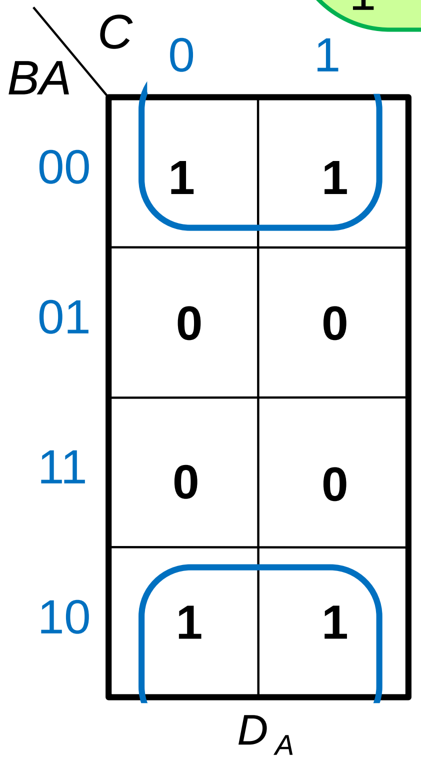
| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | 0 | 1 |
| 11 | 1 | 0 |
| 10 | 0 | 1 |

$D_C$



| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 0 |
| 01 | 1 | 1 |
| 11 | 0 | 0 |
| 10 | 1 | 1 |

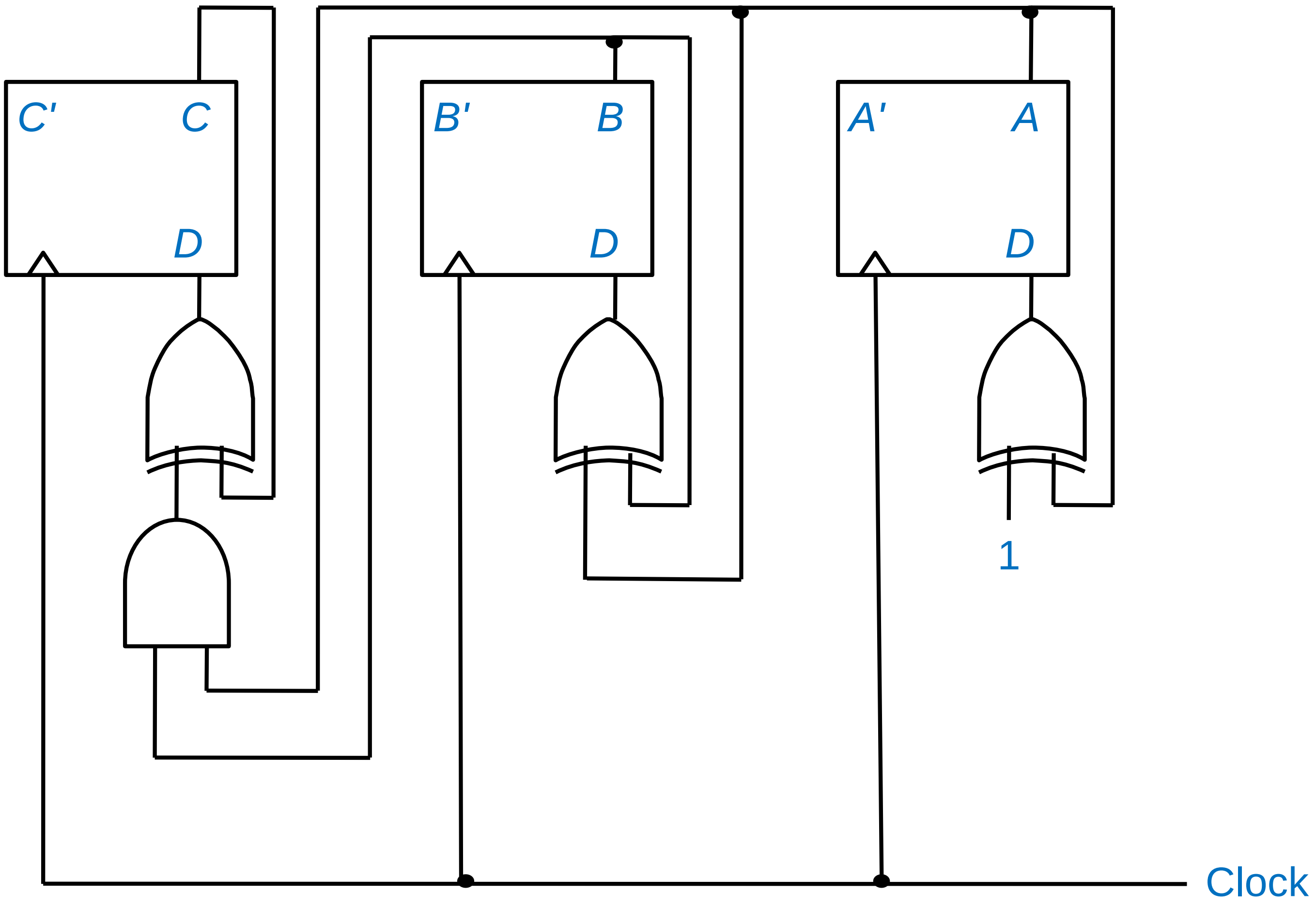
$D_B$



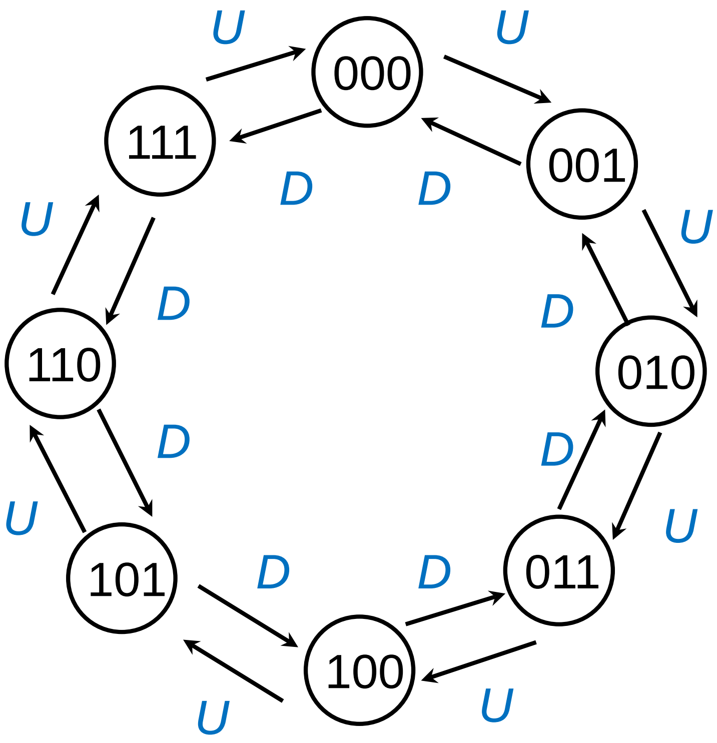
| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | 1 |
| 01 | 0 | 0 |
| 11 | 0 | 0 |
| 10 | 1 | 1 |

$D_A$

# Counting 0—7 (3/3)



# Up-Down Counter



| $CBA$ | $C^+B^+A^+$ | |
|---|---|---|
| | $U$ | $D$ |
| 000 | 001 | 111 |
| 001 | 010 | 000 |
| 010 | 011 | 001 |
| 011 | 100 | 010 |
| 100 | 101 | 011 |
| 101 | 110 | 100 |
| 110 | 111 | 101 |
| 111 | 000 | 110 |

pp.20

Do not allow:
$U = D = 1$

Up counter:
$U = 1$; $D = 0$

Down counter:
$U = 0$; $D = 1$

$D_A = A^+ = A \oplus (U + D) = A'$ (*A* changes state every clock cycle)

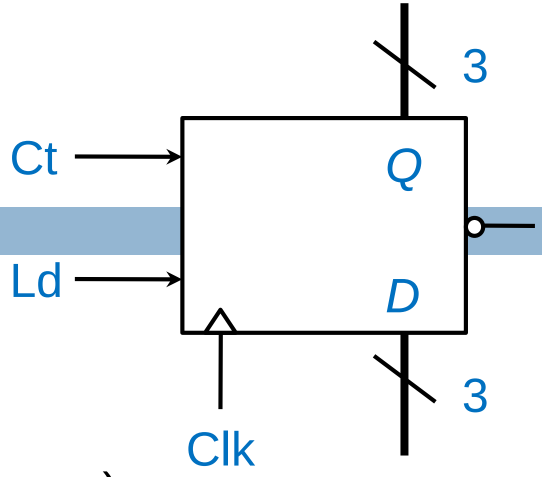
$D_B = B^+ = B \oplus (UA + DA') = B \oplus A'$ (*B* changes state when $A = 0$)

$D_C = C^+ = C \oplus (UBA + DB'A') = C \oplus B'A'$ (*C* changes state when $B = A = 0$)

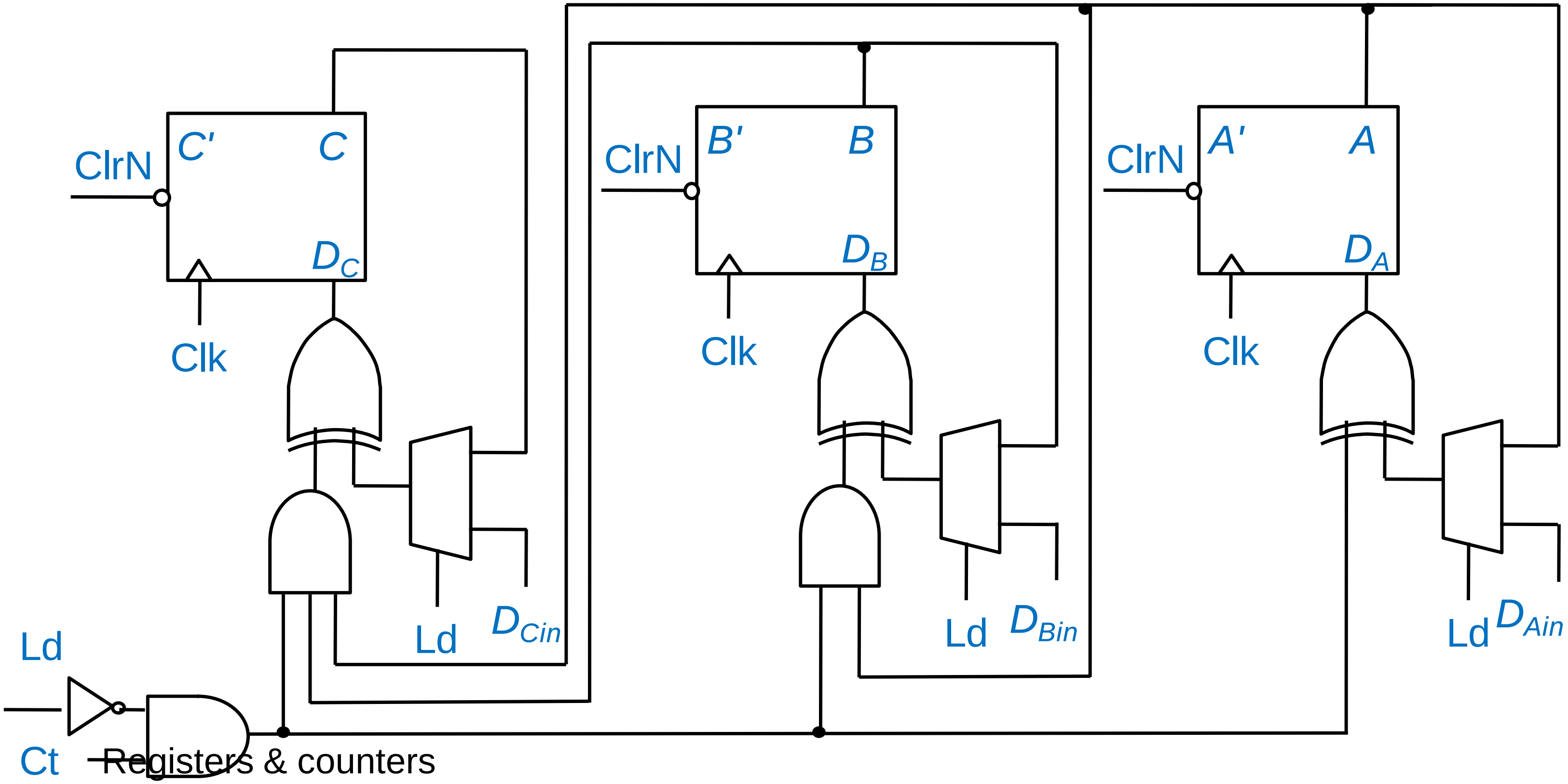
Check Figure 12-18 in the text book
for the detailed implementation

# Loadable Counter

| ClrN | Ld | Ct | $C^+$ | $B^+$ | $A^+$ | |
|---|---|---|---|---|---|---|
| 0 | X | X | 0 | 0 | 0 | |
| 1 | 1 | X | $D_C$ | $D_B$ | $D_A$ | (load) |
| 1 | 0 | 0 | $C$ | $B$ | $A$ | (no change) |
| 1 | 0 | 1 | Present state +1 | | | |



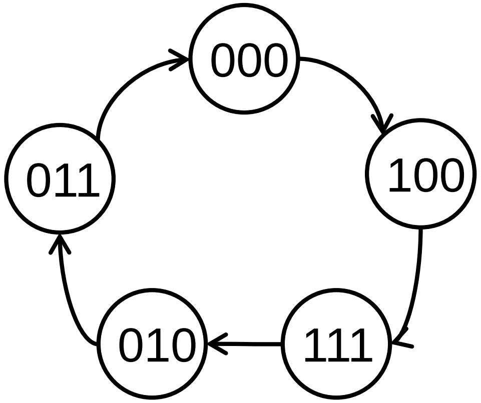
Ld: Load
Ct: Count
ClrN: Clear



# Counters for Other Sequence

# State Diagram of a Counter

- □ **What if the sequence is not in straight binary order?**



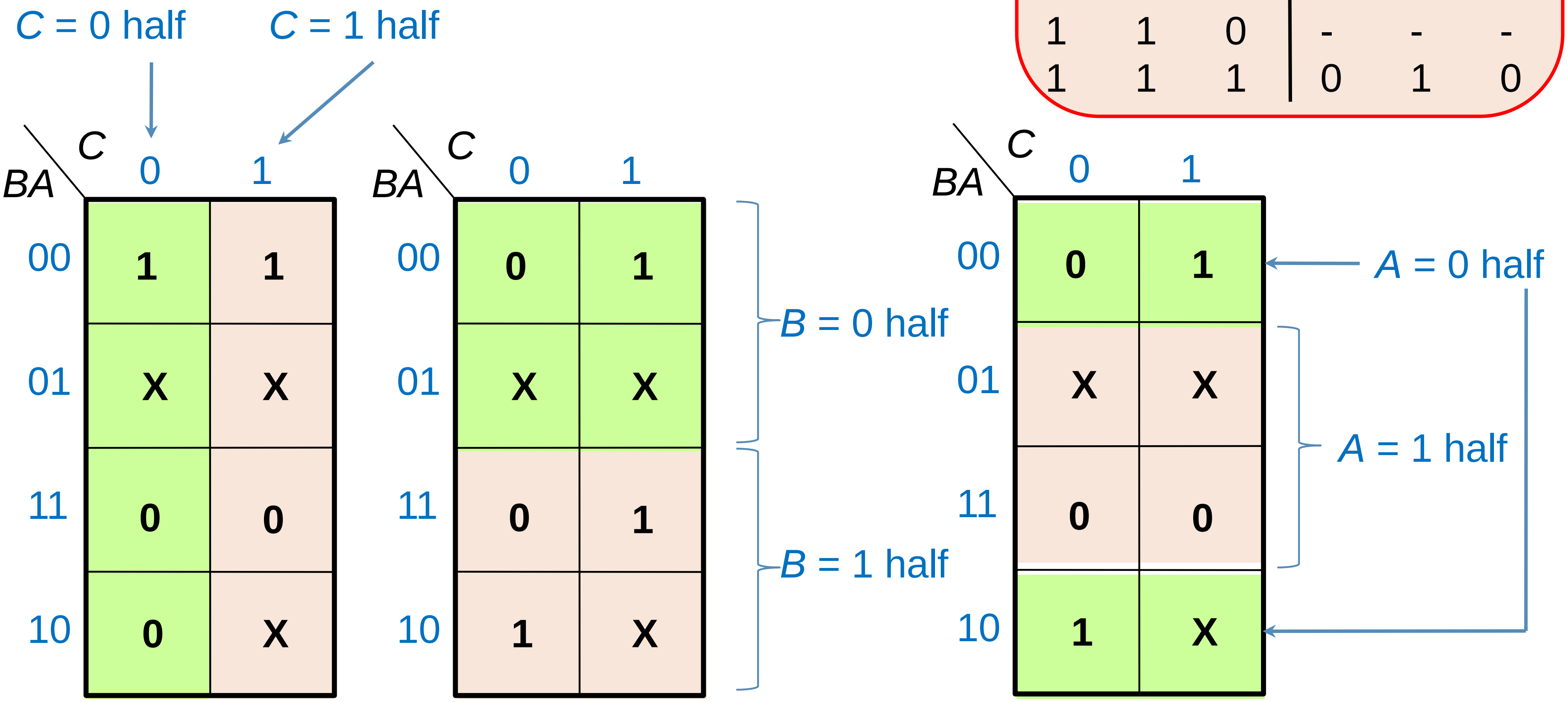
| $C$ | $B$ | $A$ | $C^+$ | $B^+$ | $A^+$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | - | - | - |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | - | - | - |
| 1 | 1 | 0 | - | - | - |
| 1 | 1 | 1 | 0 | 1 | 0 |

Unspecified ⇒ don't care

# K-Map Derivation (1/2)

- Next states

| $C$ | $B$ | $A$ | $C^+$ | $B^+$ | $A^+$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | - | - | - |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | - | - | - |
| 1 | 1 | 0 | - | - | - |
| 1 | 1 | 1 | 0 | 1 | 0 |



$C$ = 0 half

$C$ = 1 half

| $BA$ \ $C$ | 0 | 1 |
|---|---|---|
| 00 | 1 | 1 |
| 01 | X | X |
| 11 | 0 | 0 |
| 10 | 0 | X |

$C^+$

| $BA$ \ $C$ | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 0 | 1 |
| 10 | 1 | X |

$B^+$

$B$ = 0 half

$B$ = 1 half

| $BA$ \ $C$ | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 0 | 0 |
| 10 | 1 | X |

$A^+$

$A$ = 0 half

$A$ = 1 half

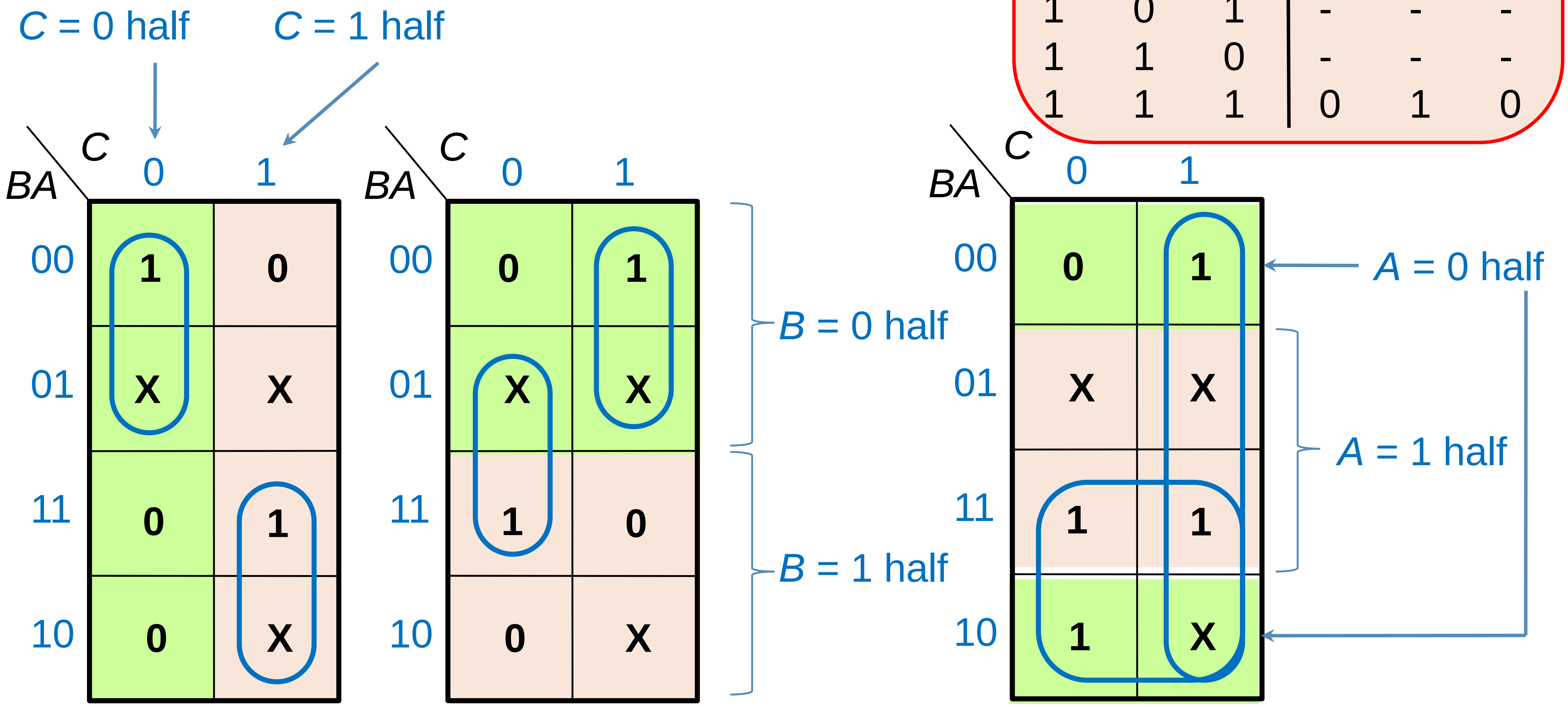
# K-Map (2/2)

- □ **T inputs: $T = Q \oplus Q^+$**

| $Q$ | $Q^+$ | $T$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$T = Q^+ \oplus Q$

| $C$ | $B$ | $A$ | $C^+$ | $B^+$ | $A^+$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | - | - | - |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | - | - | - |
| 1 | 1 | 0 | - | - | - |
| 1 | 1 | 1 | 0 | 1 | 0 |



| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | 0 |
| 01 | X | X |
| 11 | 0 | 1 |
| 10 | 0 | X |

$T_C$

$T_C = C'B' + CB$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 1 | 0 |
| 10 | 0 | X |

$T_B$

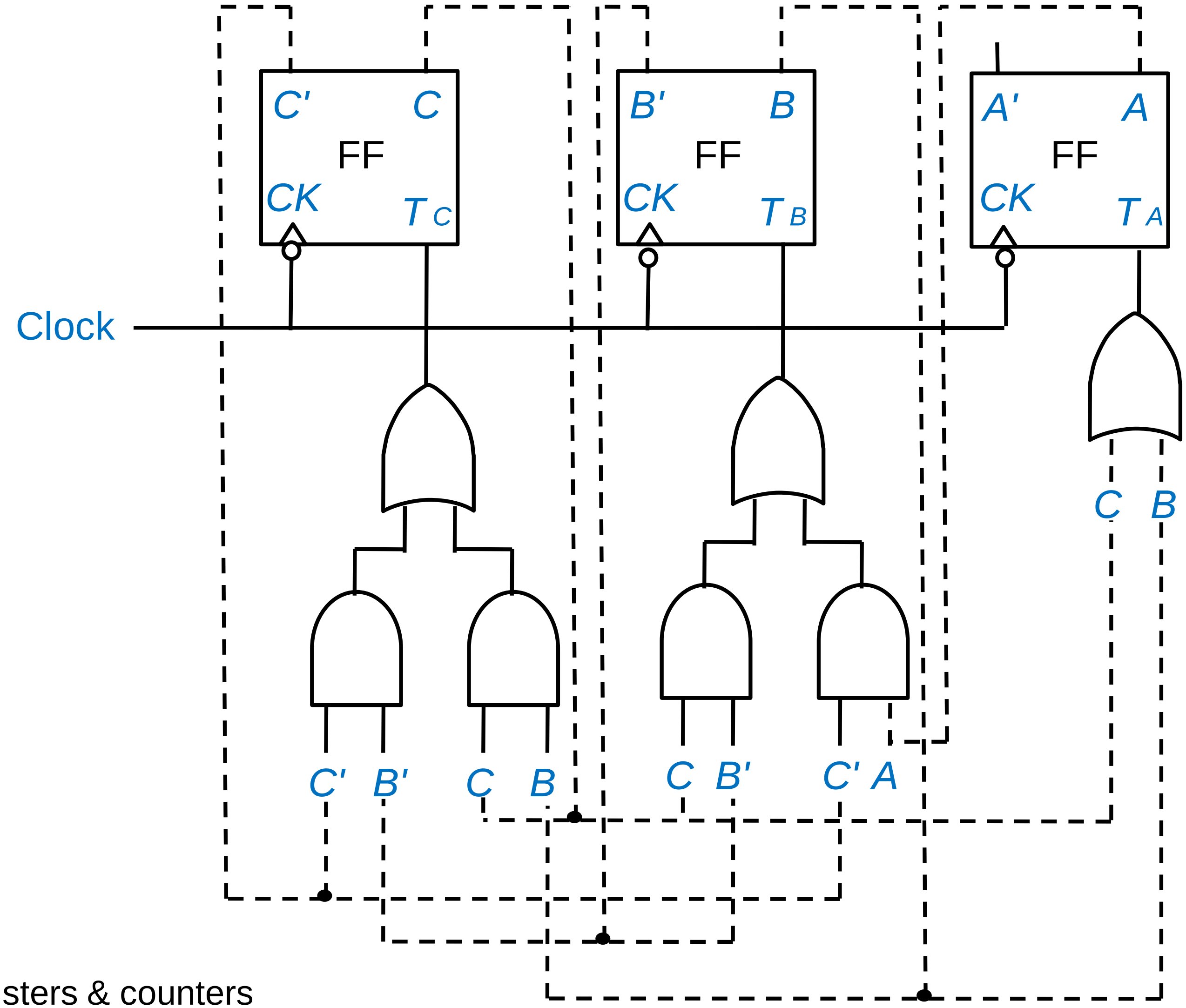
$T_B = C'A + CB'$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 1 | 1 |
| 10 | 1 | X |

$T_A$

$T_A = C + B$

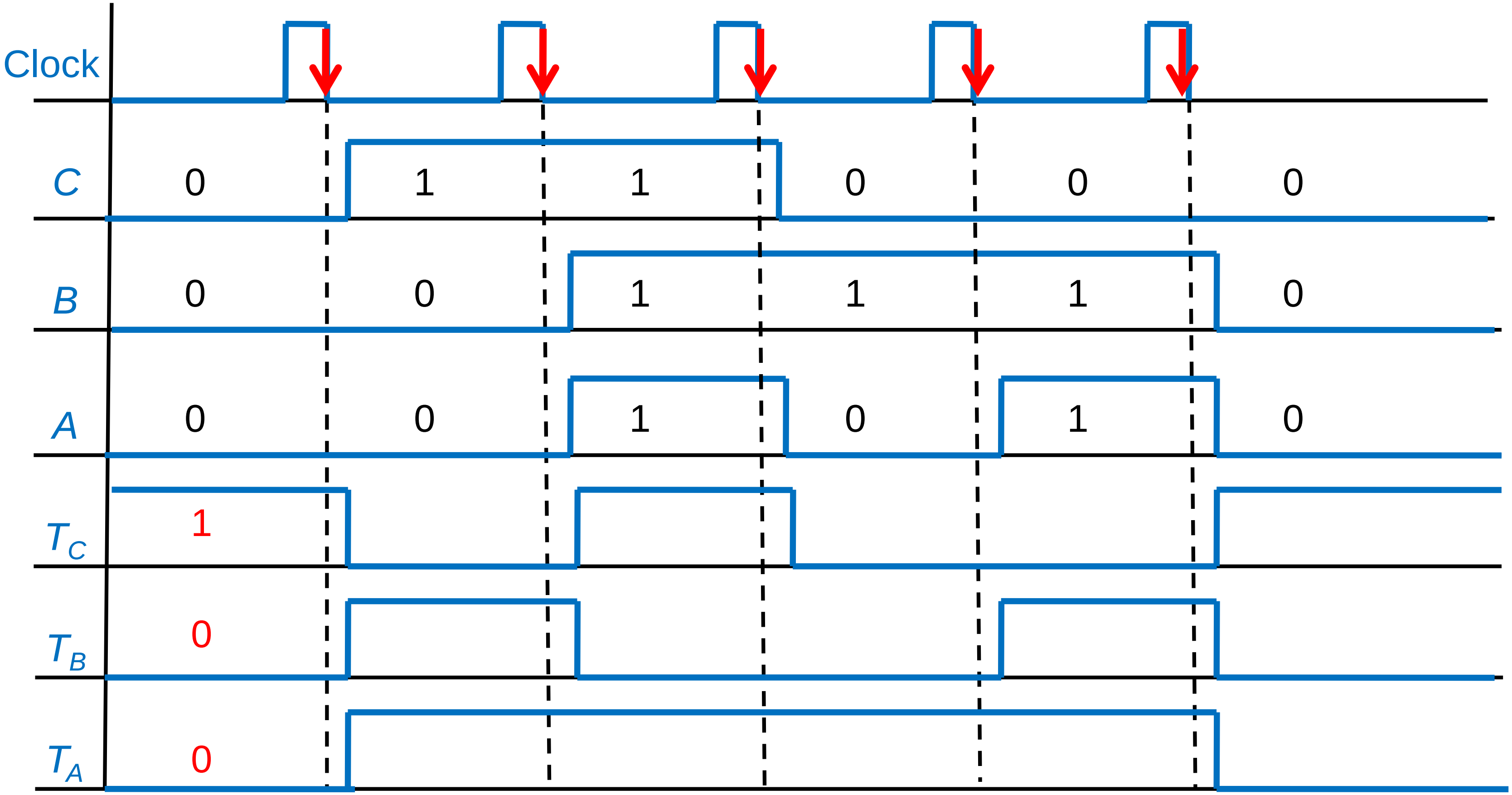
# Logic



# Timing Diagram

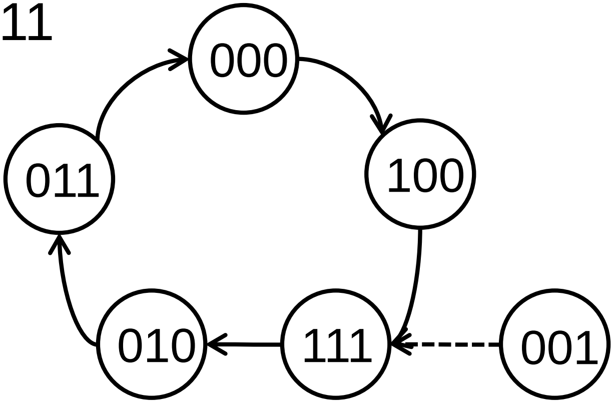
- □ **Negative-edge triggered counter**

$T_A = C + B$
$T_B = C'A + CB'$
$T_C = C'B' + CB$



# Don't Care States

- **If FFs are initially set to *CBA*=001**
  - Tracking signals through the network shows that $T_C$=$T_B$=1, so the state changes to 111



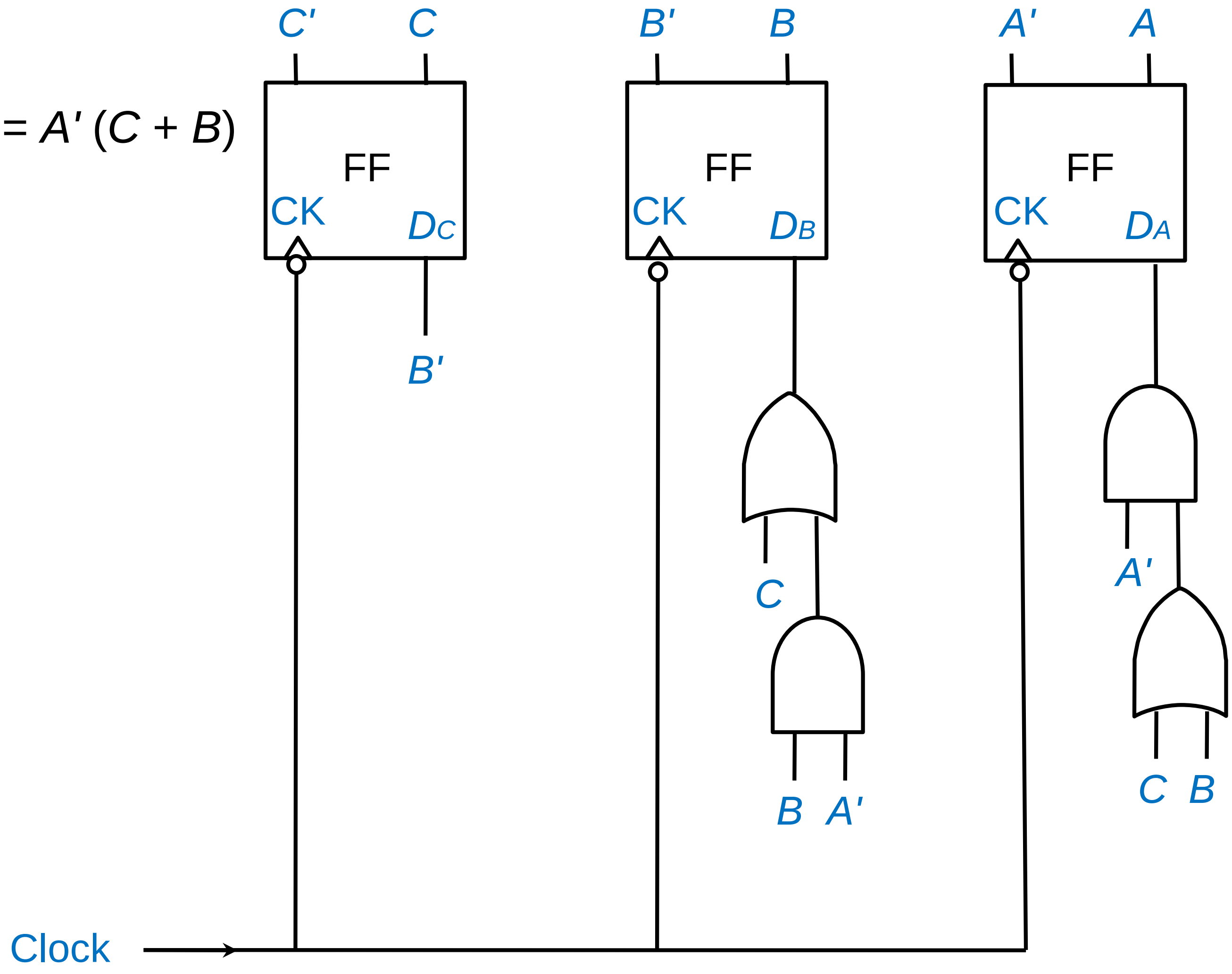
- **When the power is turned on, the initial states of all FFs are unpredictable!!**
  - Don't care states should be checked to make sure that they eventually lead into the main counting sequence
  - Or use power-up reset

# Using D FFs Instead

$D_A = A^+ = CA' + BA' = A'(C + B)$

$D_B = B^+ = C + BA'$

$D_C = C^+ = B'$



# Counter Design Using S-R & J-K FFs

# Recap S-R FFs

- **Q: What is the relation between *S*, *R* and *Q*, $Q^+$?**

| $S$ | $R$ | $Q$ | $Q^+$ | Operation |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | Unchanged |
| 0 | 0 | 1 | 1 | |
| 0 | 1 | 0 | 0 | Reset $Q$ to 0 |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 1 | Set $Q$ to 1 |
| 1 | 0 | 1 | 1 | |
| 1 | 1 | 0 | - | inputs not allowed |
| 1 | 1 | 1 | - | |

| $Q$ $Q^+$ | $S$ $R$ |
|---|---|
| 0 0 | 0 0<br>0 1 |
| 0 1 | 1 0 |
| 1 0 | 0 1 |
| 1 1 | 0 0<br>1 0 |

| $Q$ $Q^+$ | $S$ $R$ |
|---|---|
| 0 0 | 0 X |
| 0 1 | 1 0 |
| 1 0 | 0 1 |
| 1 1 | X 0 |

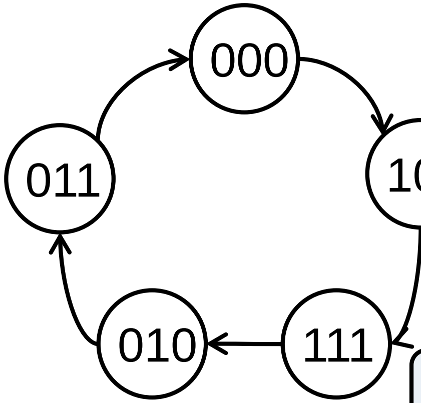
Excitation Table

- **However, actually, we do it reversely**
  - ~~$S$, $R$ ⇒ $Q$~~, $Q^+$
  - $S$, $R$ ⇐ $Q$, $Q^+$

# Using S-R FFs (1/3)

- □ **Derive S-R FF input maps from the excitation table**

| $Q$ | $Q^+$ | $S$ | $R$ |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | X | 0 |



| $C$ | $B$ | $A$ | $C^+$ | $B^+$ | $A^+$ | $S_C$ | $R_C$ | $S_B$ | $R_B$ | $S_A$ | $R_A$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | X | 0 | X |
| 0 | 0 | 1 | - | - | - | X | X | X | X | X | X |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | X | X | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | X | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | X | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | - | - | - | X | X | X | X | X | X |
| 1 | 1 | 0 | - | - | - | X | X | X | X | X | X |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | X | 0 | 0 | 1 |

# Using S-R FFs (2/3)

- □ **Derive S-R FF inputs from next state maps (faster)**

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | 1 |
| 01 | X | X |
| 11 | 0 | 0 |
| 10 | 0 | X |

$C^+$

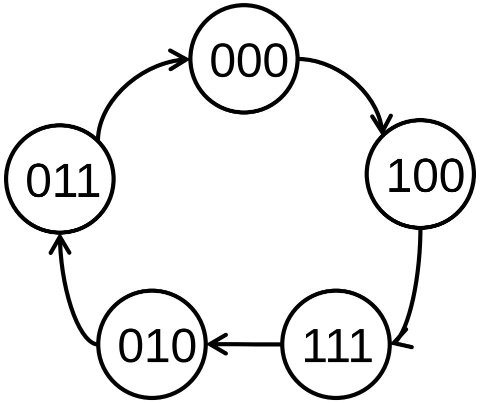
| BA \ C | 0 | 1 |
|---|---|---|
| 00 | | |
| 01 | X | X |
| 11 | X | 1 |
| 10 | X | X |

$R_C = A$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | X |
| 01 | X | X |
| 11 | | |
| 10 | | X |

$S_C = B'$

| $Q$ | $Q^+$ | $S$ | $R$ |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | X | 0 |



# Using S-R FFs (3/3)

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | 1 |
| 01 | X | X |
| 11 | 0 | 0 |
| 10 | 0 | X |

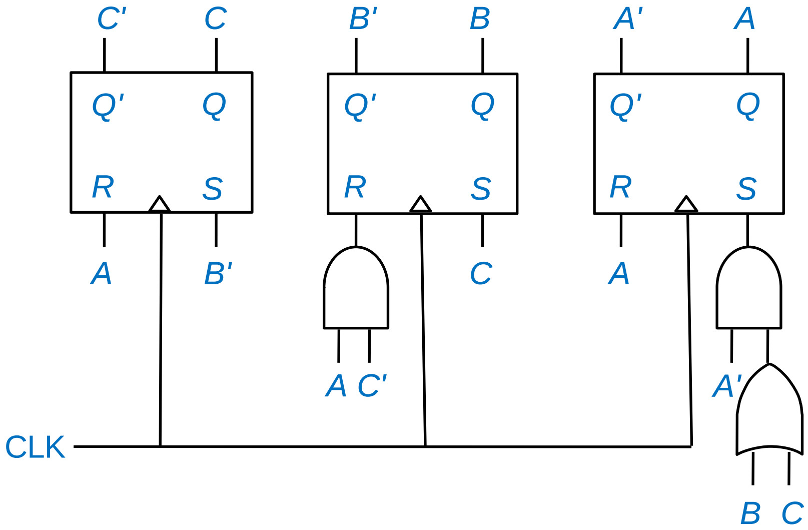
$C^+$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 0 | 1 |
| 10 | 1 | X |

$B^+$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 0 | 0 |
| 10 | 1 | X |

$A^+$



| BA \ C | 0 | 1 |
|---|---|---|
| 00 |  |  |
| 01 | X | X |
| 11 | X | 1 |
| 10 | X | X |

$R_C = A$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | X |
| 01 | X | X |
| 11 |  |  |
| 10 |  | X |

$S_C = B'$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | X |  |
| 01 | X | X |
| 11 | 1 |  |
| 10 |  | X |

$R_B = C'A$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 |  | 1 |
| 01 | X | X |
| 11 |  | X |
| 10 | X | X |

$S_B = C$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | X |  |
| 01 | X | X |
| 11 | 1 | 1 |
| 10 |  | X |

$R_A = A$

| BA \ C | 0 |  |
|---|---|---|
| 00 |  | 1 |
| 01 | X | X |
| 11 |  |  |
| 10 | 1 | X |

$S_A = CA' + BA'$
$= A'(C+B)$

# Using J-K FFs (1/2)

| $J$ | $K$ | $Q$ | $Q^+$ | Operation |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | Unchanged |
| 0 | 0 | 1 | 1 | |
| 0 | 1 | 0 | 0 | Reset $Q$ to 0 |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 1 | Set $Q$ to 1 |
| 1 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 1 | Toggle |
| 1 | 1 | 1 | 0 | |

| $Q$ | $Q^+$ | $J$ | $K$ |
|---|---|---|---|
| 0 | 0 | 0<br>0 | 0<br>1 |
| 0 | 1 | 1<br>1 | 0<br>1 |
| 1 | 0 | 0<br>1 | 1<br>1 |
| 1 | 1 | 0<br>1 | 0<br>0 |

| $Q$ | $Q^+$ | $J$ | $K$ |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | X |
| 1 | 0 | X | 1 |
| 1 | 1 | X | 0 |

| $C$ | $B$ | $A$ | $C^+$ | $B^+$ | $A^+$ | $J_C$ | $K_C$ | $J_B$ | $K_B$ | $J_A$ | $K_A$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | X | 0 | X | 0 | X |
| 0 | 0 | 1 | - | - | - | X | X | X | X | X | X |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | X | X | 0 | 1 | X |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | X | X | 1 | X | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | X | 0 | 1 | X | 1 | X |
| 1 | 0 | 1 | - | - | - | X | X | X | X | X | X |
| 1 | 1 | 0 | - | - | - | X | X | X | X | X | X |
| 1 | 1 | 1 | 0 | 1 | 0 | X | 1 | X | 0 | X | 1 |

# Using J-K FFs (2/2)

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | 1 |
| 01 | X | X |
| 11 | 0 | 0 |
| 10 | 0 | X |

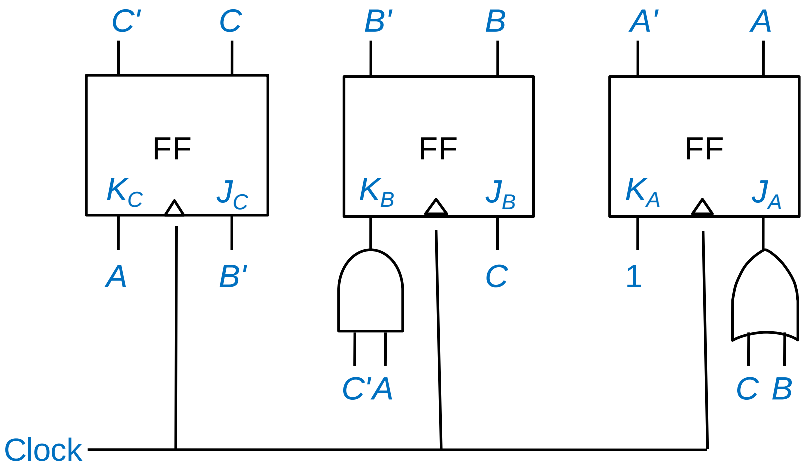
$C^+$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 0 | 1 |
| 10 | 1 | X |

$B^+$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | X | X |
| 11 | 0 | 0 |
| 10 | 1 | X |

$A^+$



| BA \ C | 0 | 1 |
|---|---|---|
| 00 | 1 | X |
| 01 | X | X |
| 11 |  | X |
| 10 |  | X |

$J_C = B'$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | X |  |
| 01 | X | X |
| 11 | X | 1 |
| 10 | X | X |

$K_C = A$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 |  | 1 |
| 01 | X | X |
| 11 | X | X |
| 10 | X | X |

$J_B = C$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | X | X |
| 01 | X | X |
| 11 | 1 |  |
| 10 |  | X |

$K_B = C'A$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 |  | 1 |
| 01 | X | X |
| 11 | X | X |
| 10 | 1 | X |

$J_A = C + B$

| BA \ C | 0 | 1 |
|---|---|---|
| 00 | X | X |
| 01 | X | X |
| 11 | 1 | 1 |
| 10 | X | X |

$K_A = 1$

# Derivation of FF Input Equations

## Summary

# Derivation of Flip-Flop Input Equations

- **Determine the FF input equations from the next-state equations using K-maps**
  - Always copy X’s from next state maps onto input maps first
  - Fill in the remaining squares with Q=0 and Q=1, separately

| Type of F/F | Input | Q=0 | | Q=1 | | Rules for forming input map from next state map | |
|---|---|---|---|---|---|---|---|
| | | $Q^+=0$ | $Q^+=1$ | $Q^+=0$ | $Q^+=1$ | Q=0 Half of Map | Q=1 Half of Map |
| D | D | 0 | 1 | 0 | 1 | No change | No change |
| T | T | 0 | 1 | 1 | 0 | No change | Complement |
| S-R | S | 0 | 1 | 0 | x | No change | Replace 1’s with x’s |
| | R | x | 0 | 1 | 0 | Replace 0’s with x’s<br>Replace 1’s with 0’s | Complement |
| J-K | J | 0 | 1 | x | x | No change | Fill in with x’s |
| | K | x | x | 1 | 0 | Fill in with x’s | Complement |

# Important Tables

| $Q$ | $Q^+$ | $D$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

D FF

| $Q$ | $Q^+$ | $T$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Toggle FF

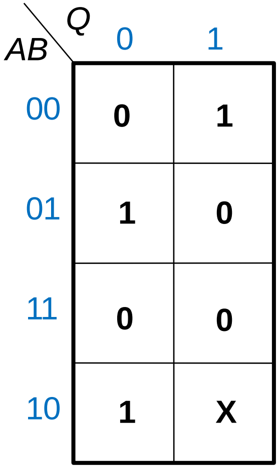
| $Q$ | $Q^+$ | $S$ | $R$ |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | X | 0 |

SR FF

| $Q$ | $Q^+$ | $J$ | $K$ |
|---|---|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | X |
| 1 | 0 | X | 1 |
| 1 | 1 | X | 0 |

JK FF

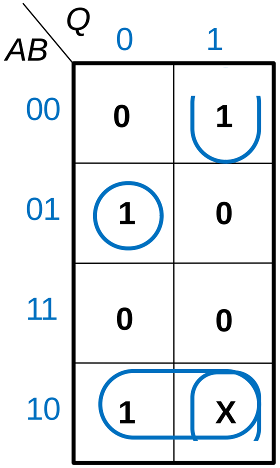
# Example



| AB \ Q | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | 1 | 0 |
| 11 | 0 | 0 |
| 10 | 1 | X |

$Q^+$

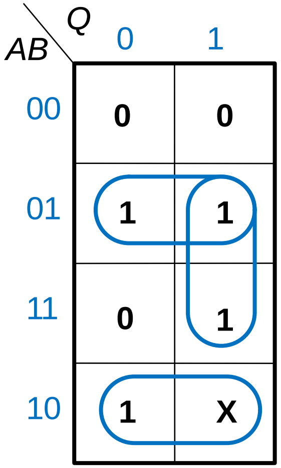
Next-state map



| AB \ Q | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | 1 | 0 |
| 11 | 0 | 0 |
| 10 | 1 | X |

$D = Q'A'B + QB' + AB'$

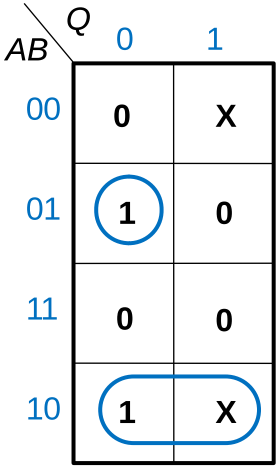
*D* input map



| AB \ Q | 0 | 1 |
|---|---|---|
| 00 | 0 | 0 |
| 01 | 1 | 1 |
| 11 | 0 | 1 |
| 10 | 1 | X |

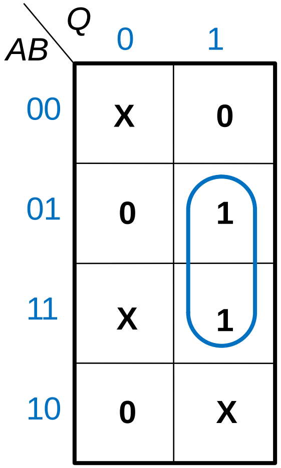
$T = A'B + AB' + QB$

*T* input map



| AB \ Q | 0 | 1 |
|---|---|---|
| 00 | 0 | X |
| 01 | 1 | 0 |
| 11 | 0 | 0 |
| 10 | 1 | X |

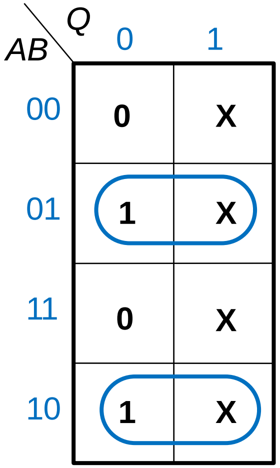
$S = AB' + Q'A'B$



| AB \ Q | 0 | 1 |
|---|---|---|
| 00 | X | 0 |
| 01 | 0 | 1 |
| 11 | X | 1 |
| 10 | 0 | X |

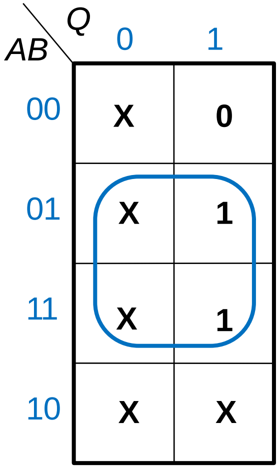
$R = QB$

*S-R* input maps



| AB \ Q | 0 | 1 |
|---|---|---|
| 00 | 0 | X |
| 01 | 1 | X |
| 11 | 0 | X |
| 10 | 1 | X |

$J = A'B + AB'$



| AB \ Q | 0 | 1 |
|---|---|---|
| 00 | X | 0 |
| 01 | X | 1 |
| 11 | X | 1 |
| 10 | X | X |

$K = B$

*J-K* input maps

# Example: 4-Variable Maps (1/3)

□ **4-variable maps (*A*, *B*, *C*, $Q_i$) ) using 3 different type FFs**

| $BC \backslash Q_1A$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 1 |
| 01 | X | 1 | 1 | 0 |
| 11 | 1 | X | X | 1 |
| 10 | 0 | 0 | 0 | X |

$Q_1$ = 0 half $Q_1$ = 1 half

$Q_1^+$

| $BC \backslash Q_1A$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 | 0 |
| 01 | X | 1 | 0 | 1 |
| 11 | 1 | X | X | 0 |
| 10 | 0 | 0 | 1 | X |

$T_1$

| $Q$ $Q^+$ | $T$ |
|---|---|
| 0 0 | 0 |
| 0 1 | 1 |
| 1 0 | 1 |
| 1 1 | 0 |

Toggle

# Example (2/3)

$Q_2$=0 half

$Q_2$ =1 half

| $CQ_2$ \ $AB$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | X | 1 | 0 |
| 01 | 0 | 0 | X | 1 |
| 11 | 1 | 0 | X | 1 |
| 10 | X | 0 | 0 | 1 |

$Q_2{}^+$

| $CQ_2$ \ $AB$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | X | 1 | 0 |
| 01 | 0 | 0 | X | X |
| 11 | X | 0 | X | X |
| 10 | X | 0 | 0 | 1 |

$S_2$

| $CQ_2$ \ $AB$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | X | 0 | X |
| 01 | 1 | 1 | X | 0 |
| 11 | 0 | 1 | X | 0 |
| 10 | X | X | X | 0 |

$R_2$

| $Q$ $Q^+$ | $S$ $R$ |
|---|---|
| 0 0 | 0 X |
| 0 1 | 1 0 |
| 1 0 | 0 1 |
| 1 1 | X 0 |

Set-Reset

# Example (3/3)

| $Q_3C$ \ $AB$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | X |
| 01 | 0 | 1 | X | 1 |
| 11 | X | X | 0 | 0 |
| 10 | 1 | 1 | 1 | 0 |

$Q_3$=0 half (rows 00, 01)

$Q_3$=1 half (rows 11, 10)

$Q_3^{\ +}$

| $Q_3C$ \ $AB$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | X |
| 01 | 0 | 1 | X | 1 |
| 11 | X | X | X | X |
| 10 | X | X | X | X |

$J_3 = A + BC$

| $Q_3C$ \ $AB$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | X | X | X | X |
| 01 | X | X | X | X |
| 11 | X | X | 1 | 1 |
| 10 | 0 | 0 | 0 | 1 |

$K_3 = C + AB'$

| $Q$ $Q^+$ | $J$ $K$ |
|---|---|
| 0 0 | 0 X |
| 0 1 | 1 X |
| 1 0 | X 1 |
| 1 1 | X 0 |

Jump-Clear