

1. A combinational circuit has four binary inputs (A, B, C, D) and three binary outputs (X, Y, Z).
 XYZ represents a binary number whose value equals the number of 0s / 1s at the input.
 And if four inputs are the same, all outputs are don't care.
 For example, if ABCD = 1011 then XYZ = 001/011.

(a) (24%) Complete the truth table for the circuit.

Use X to represent don't care.

(b) (15%) Find the minterm expansion for Y using m -notation.

(c) (15%) Find the maxterm expansion for Z using M -notation.

Note that the order of the input variables for m -notation and M -notation is ABCD.

(a)

ABCD	X	Y	Z
0000	X / X	X / X	X / X
0001	0 / 0	1 / 0	1 / 1
0010	0 / 0	1 / 0	1 / 1
0011	0 / 0	1 / 1	0 / 0
0100	0 / 0	1 / 0	1 / 1
0101	0 / 0	1 / 1	0 / 0
0110	0 / 0	1 / 1	0 / 0
0111	0 / 0	0 / 1	1 / 1
1000	0 / 0	1 / 0	1 / 1
1001	0 / 0	1 / 1	0 / 0
1010	0 / 0	1 / 1	0 / 0
1011	0 / 0	0 / 1	1 / 1
1100	0 / 0	1 / 1	0 / 0
1101	0 / 0	0 / 1	1 / 1
1110	0 / 0	0 / 1	1 / 1
1111	X / X	X / X	X / X

(b) each term is 1 point, don't care part is 5 points.

$$Y = \sum m(1,2,3,4,5,6,8,9,10,12) + \sum d(0,15)$$

$$Y = \sum m(3,5,6,7,9,10,11,12,13,14) + \sum d(0,15)$$

(c) each term is 1 point, don't care part is 5 points.

$$Z = \prod M(3,5,6,9,10,12) * \prod D(0,15)$$

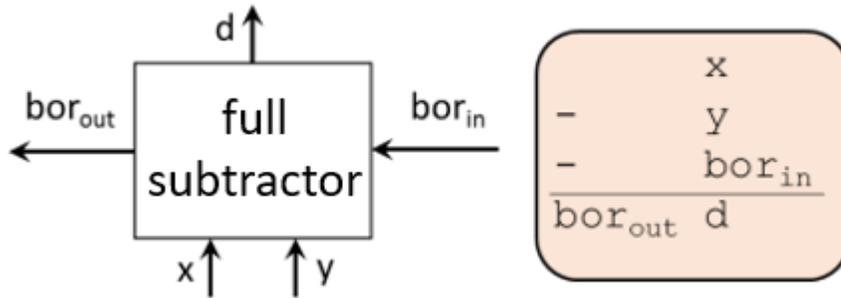
$$Z = \prod M(3,5,6,9,10,12) * \prod D(0,15)$$

2. A full subtractor has three binary inputs (x , y , bor_{in}) and two outputs (d , bor_{out}).

(a) (16%) Complete the truth table for a full subtractor.

(b) (15%) Find the minterm expansions for d / bor_{out} .

(c) (15%) Find the maxterm expansions for bor_{out} / d .



(a)

x	y	bor_{in}	d	bor_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

(b) Each term is 2 points.

$$d = \sum m(1, 2, 4, 7)$$

$$= x'y'\text{bor}_{in} + x'y\text{bor}_{in}' + xy'\text{bor}_{in}' + xy\text{bor}_{in}$$

$$\text{bor}_{out} = \sum m(1, 2, 3, 7)$$

$$= x'y'\text{bor}_{in} + x'y\text{bor}_{in}' + x'y\text{bor}_{in} + xy\text{bor}_{in}$$

(c) Each term is 2 points.

$$\text{bor}_{out} = \prod M(0, 4, 5, 6)$$

$$= (x + y + \text{bor}_{in})(x' + y + \text{bor}_{in})(x' + y + \text{bor}_{in}')(x' + y' + \text{bor}_{in})$$

$$d = \prod M(0, 3, 5, 6)$$

$$= (x + y + \text{bor}_{in})(x + y' + \text{bor}_{in}')(x' + y + \text{bor}_{in}')(x' + y' + \text{bor}_{in})$$